## ABERRATIONS

### Chromatic Aberration

The presence of material dispersion causes the refractive index to vary with wavelength. This means that strictly speaking the results of the paraxial approximation are only true for the central design wavelength, since the expression for thin lens power (Theory Pg 2) contains the refractive index.

The first two chromatic errors to appear are a variation with wavelength of the paraxial image plane position and image height. These are known respectively as longitudinal and lateral chromatic aberration. The accompanying figures show the behavior of a typical lens.

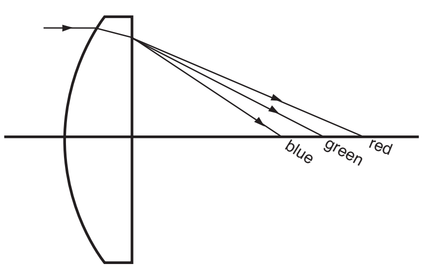
A useful parameter in assessing the magnitude of chromatic aberration is the Abbe number (V–value). This is defined in terms of the refractive indices $n_1$,$n_2$, and $n_3$ at three wavelengths $\lambda_1$,$\lambda_2$ and $\lambda_3$ (the design wavelength and the long and short wavelength limits of the spectral band respectively) by

$$V = \frac{n_1 - 1}{n_3 - n_2}$$

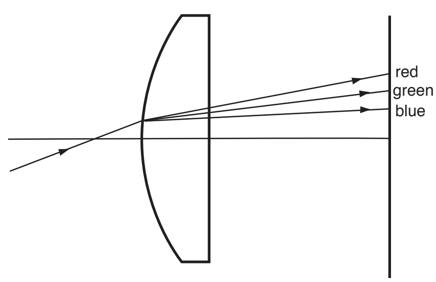
The fractional change in focal length over the spectral range of interest is given by 1/V. So for BK7 lenses used between the wavelengths 486.1nm and 656.6nm where the value is $V_d$=64.17, the change in focal length is approximately 1.6 per cent.

If the chief ray passes centrally through the lens ( i.e. the lens is at the stop) then it is undeflected and the contribution to lateral chromatic aberration is zero. It is worth pointing out that lateral chromatic aberration is a shift in image height with the image plane fixed at the position given for the design wavelength.

Changes of lens shape have no influence on the chromatic aberration to a first order approximation, and at least two components of different materials are required for its correction. There are exceptions such as the two lens Dialyte design and binary lenses which combine refracting and diffracting power, but these are beyond the level of discussion here.




Longitudinal Chromatic Aberration

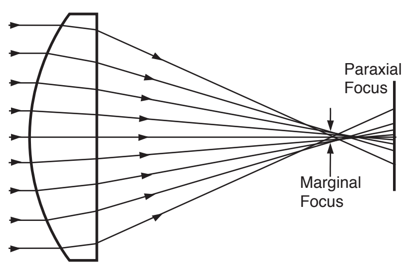



Lateral Chromatic Aberration

**Seidel Aberrations**

The most important aberrations in the majority of applications are the Seidel (also known as Primary or Third order) aberrations. These are the first aberrations to have an impact on the image quality as the aperture and field angles are increased beyond the point at which the paraxial approximation ceases to remain accurate. There are five Seidel monochromatic aberrations. In addition there are two Primary chromatic aberrations. These are the most significant in the majority of applications.

**Primary Spherical Aberration**




The figure shows the situation for an infinitely distant object conjugate. As the incident ray height at the lens is increased, the angle of incidence also increases. The deflection of the ray at that boundary increases more rapidly than the amount predicted by the paraxial approximation.

The net effect is for a lens with spherical surfaces to exhibit excess power for rays farther from the optical axis, bringing these rays to a focus closer to the lens. The difference between the paraxial and marginal focus positions is called the longitudinal spherical aberration.

A more convenient measure of the magnitude of the aberration in most cases is to examine the distance from the axis at which rays pierce a particular focal plane. This image criterion is the transverse spherical aberration. In the Seidel approximation, the ray error is proportional to the 3rd power of the initial ray height.

A smaller spot size can be obtained by selecting an appropriate focal plane. In the geometrical optics approximation (i.e. ignoring diffraction effects) and in the absence of higher order aberrations, the optimum plane lies 3/4 of the way from the paraxial focus to the marginal focus. This results in a spot size which is 1/4 of the size which would be obtained at the paraxial focus position.

If the system is diffraction limited then the wavefront is a more reliable indicator of image quality. The wavefront error associated with primary spherical aberration is dependent on the 4th power of the aperture. The optimum focal plane in this situation would be midway between the paraxial and marginal focal positions. This location maximizes the intensity at the center of the diffraction spot.

A reduction in the amount of spherical aberration can be achieved in several ways. The simplest method is to reduce the aperture, and here the improvement can be quite dramatic. For example, a reduction of 20% in lens aperture halves the geometrical blur size and reduces the wavefront error to 40% of its initial value.

Another technique of reducing the spherical aberration is to change the shape of the lens, a technique known as “bending”. The spherical aberration of a lens is usually reduced if the deflections of the ray which occur at the two surfaces are made more nearly equal. Depending on the conjugates, i.e. the magnification, there is an optimum shape and orientation for the lens.
By extending this technique to the use of more than one lens in a system, it can be seen that sharing the power between the surfaces of several lenses results in a reduction of the magnitude of the spherical aberration.

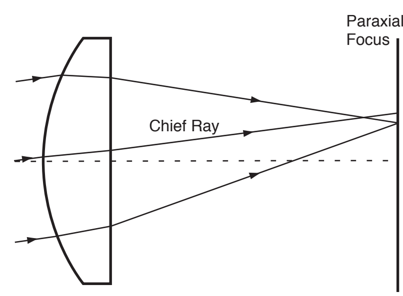
With glass of refractive index close to 1.5 the best positive lens form for use with infinite conjugates is a bi–convex lens with a 6:1 ratio in its radii of curvature, with the steepest side positioned towards the infinite conjugate. However the performance gain over a plano–convex lens is minimal and for reasons of cost the plano–convex form is nearly always used for this application.

For 1:1 imaging, the equi–convex lens is the favourable shape as it produces an almost equal angular deflection of the ray path per surface.

The final technique for the reduction in spherical aberration involves introducing lenses of opposite power, which have controlled amounts of spherical aberration of opposing signs. An example of such a case is the cemented achromatic doublets covered in a later section.

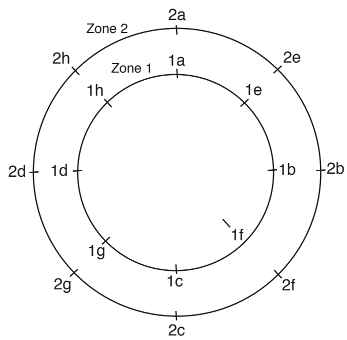
In complex lens systems, one or more of these techniques may be applied to correct the overall lens assembly. In addition surfaces with non–spherical form, or materials with controlled departures of the refractive index from the homogeneous case, may be utilized.

**Primary Coma**

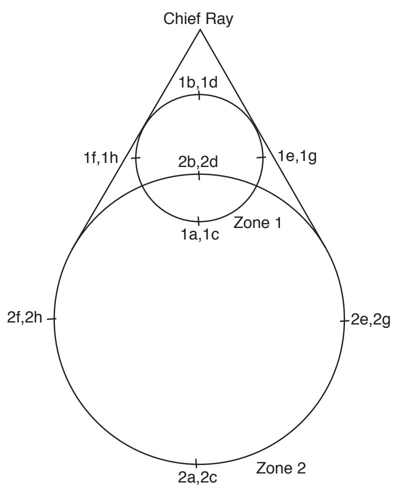



Coma is the first of the lens aberrations to appear as the conjugate points are moved away from the optical axis.
In the figure, a parallel input beam is shown approaching a plano–convex lens at an oblique angle. The ray at the upper edge of the lens has a higher angle of incidence with the curved surface than the ray at the lower edge. By analogy with the case of spherical aberration described previously, the deflection of the upper ray will be greater, and it will intersect the chief ray closer to the lens than the ray from the lower edge.

## ABERRATIONS




Rays in Lens Pupil




Rays at Image Plane

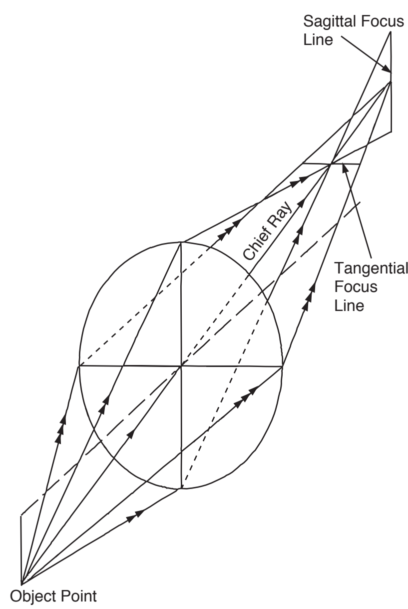
As shown in the figure above, rays passing through any circular zone of the lens pass through the paraxial image plane in a circular pattern. The center of this pattern experiences an increasing lateral shift away from the point of intersection of the principal ray with the paraxial image plane. In addition the radius of each circle increases as the selected zone at the lens is increased in diameter. Input rays at diametrically opposite points in the lens pupil fall onto the same point in the paraxial image plane.

It is the comet–like appearance of the image associated with this aberration that gives rise to the name Coma. The magnitude of the primary coma can be reduced by stopping down the lens or by choosing an appropriate bending for the lens in a similar way to the case of spherical aberration. However, for primary coma , the transverse aberration and wavefront error vary as the 2nd and 3rd powers of the ray height respectively, so the gain is not so dramatic. The optimum shape of the lens to reduce coma is such as to produce a pseudo–symmetry to the incident beam and is quite close to that which minimizes spherical aberration.

An additional method of reducing coma is to move the stop, so that the off–axis beam translates laterally across the lens and takes up a position where the deflections at the top and bottom are approximately equal. Equations given in a later section will indicate that the reduction of the coma is only possible where there is residual spherical aberration in the system.

It is worth commenting that complex lens arrangements, which exhibit a degree of symmetry about the stop, are usually substantially free from coma.

### Astigmatism and Field Curvature




Astigmatism and Field Curvature are the aberrations from which a lens suffers if it is used off–axis at a low aperture. For single thin lenses the magnitude of these aberrations is proportional to the lens power.

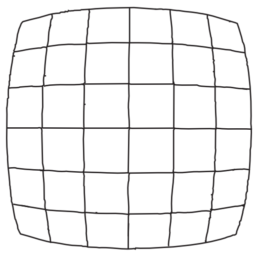
Two principal sections occur - the tangential (or meridional) plane, which contains the object point and the optical axis, and the sagittal (or radial) plane which passes through the object point and is perpendicular to the tangential plane. For the simple case shown in the figure, the rays in the tangential plane focus closer to the lens than those in the sagittal plane, the discrepancy in the focus position being known as astigmatism.

In the absence of other types of aberration, the sagittal and tangential foci are line foci, while at other planes the shape of the beam is of an elliptical cross–section. Midway between the two line foci lies the medial surface, where the spot shape is circular. This surface is the optimum choice for many applications.
The shift in focus position is proportional to the square of the image height for primary astigmatism, the positions of the S and T foci mapping out two paraboloidal surfaces as the field position is varied.
Astigmatism can be reduced by varying the stop position, provided that spherical aberration or coma is non–zero. For a lens corrected for all three image defects a multi–component system is almost always required, the shape of the components being in most cases critical and different from those of standard catalog lenses.

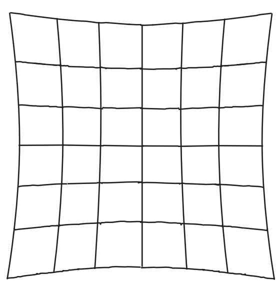
Field curvature also results in the foci of off–axis points falling on a paraboloidal surface known as the Petzval surface. It is not possible to remove this aberration by choosing a different stop position or by changing the bending of a single lens. In some cases a compromise may be found if astigmatism of opposing sign is introduced.
Adjusting the detector position by scanning or by curving a photo–sensitive material can be used in some cases, as field curvature by itself does not degrade the imagery at any individual field point.

In general the field curvature can only be reduced by using several lenses of opposing powers. This is usually only beneficial if a complex lens is used which is also well corrected for astigmatism.

### Distortion



Barrel Distortion



Pincushion Distortion

Distortion is produced when the chief rays intersect the image surface at heights different from those predicted by the paraxial approximation. The cases of pincushion and barrel distortion are shown in the figure above. The dependence of distortion on stop position is strong. A thin lens placed at the stop will exhibit no distortion, and this is also the case for a lens arrangement which is symmetrical about the stop.

In most applications involving simple catalog components, the degradation of the image quality due to astigmatism and field curvature will have reached unacceptable levels before distortion becomes a problem.

Primary distortion is proportional to the 3rd power of the field height. In certain applications it may be compensated for by scanning the image by means of micro–positioning equipment or by pre–distortion of the object, such as might be carried out on a CRT using appropriate deflection voltages.

### Computing Approximate Seidel Aberrations

It is sometimes useful to be able to get an approximate value for the magnitude of the aberrations without using real (finite) raytracing equations.
When using simple components the Seidel approximation is often sufficiently accurate. Here, at least, if higher order aberrations are present, they are usually accompanied by Seidel aberrations of a high magnitude. For this reason expressions are given for the evaluation of the Seidel aberration coefficients of thin lenses.
Most simple catalog components, can be considered as thin lenses for the purpose of evaluating the aberrations. If they are thick relative to their focal length, they are likely to be operating at a fast F/No. In this case the higher order aberrations need to be considered as well.
Even in cases where the Seidel approximation is not accurate, their computation can identify situations where the lens choice is obviously inadequate for the task and point out potential problem areas in an imaging system.

### Equations for a thin lens at the stop

To compute the Seidel coefficients for a thin lens it is necessary to know the following parameters
**1)** $a_0$, $a_1$, $a_2$, $b_0$ and $b_1$ which may be found listed in the text of each lens type. The values are sufficiently accurate for work in the region of the design wavelength.
**2)** The height of the paraxial ray h, for the on–axis object point at the lens in question.
**3)** The Lagrange Invariant H.
**4)** The value of the Abbe Number V and the refractive index n, for the wavelength region in question, if the magnitude of the chromatic aberration is to be assessed.
**5)** The conjugate parameter C which is computed from the magnification m (for that component alone), using

$$C = \frac{m+1}{m-1}$$

The Seidel coefficients then become:

$$S_1 = h^4K^3(a_0+a_1C+a_2C^2)$$

$$S_2 = -h^2H(b_0+b_1C)$$

$$S_3 = H^2K$$

$$S_4 = \frac{H^2K}{n}$$

$$S_5 = 0$$

$$C_1 = \frac{h^2K}{V}$$

$$C_2 = 0$$

### Equations for a thin lens located away from the stop - Stop shift equations

For lenses located away from the stop, the equations are slightly more complex. They require a knowledge of the height $\bar{h}$ at the lens of the paraxial chief ray from the object point of interest. It also passes through the center of the stop. This is incorporated in the term E, given by

$$E = \frac{\bar{h}}{h}$$

The primed quantities ($S_1'$ etc.) refer to the coefficients obtained if the lens were at the stop.

$$S_1 = S'_1$$

$$S_2 = S'_2+ES'_1$$

$$S_3 = S'_3+2ES'_2+E2S'_1$$

$$S_4 = S'_4$$

$$S_5 = S'_5+E(3S'_3+S'_4)+3E2S'_2+E3S'_1$$

$$C_1 = C'_1$$

$$C_2 = C'_2+EC'_1$$

## ABERRATIONS

**Example 7**

As an example of the calculation of the approximate performance of a lens system using the equations on the previous Page, we will apply them to the lens combination used in Example 5 on Page 7.

For lens 1,
(from Example 5)

$m = u_1/u_0$
$= 0$, giving a conjugate parameter
$C = -1.$
$h = 7.5$
$K = 1/75$
$H = -0.75$
$\bar{h} = 0$

$n = 1.5168$
$V = 64.17$
$a_0 = 4.3238$
$a_1 = 3.2107$
$a_2 = 1.0796$
$b_0 = 1.6053$
$b_1 = 1.3296$

Substituting these values into the equations on the previous Page, we obtain

$S_1=0.016441$ $S_2=0.002068$
$S_3=0.0075$ $S_4=0.004945$
$S_5=0.0$ $C_1=0.0011688$
$C_2=0.0$

For lens 2
(from example 5)

$m = u_2/u_1$
$= 2/3$, giving a conjugate parameter
$C = -5.$
$h=3.75$
$K=1/75$
$H=-0.75$
$\bar{h}=3.75$

The values of $n$,$V$,$a_0$, $a_1$, $a_2$, $b_0$ and $b_1$ are the same as for lens 1 as it is identical in material and form.
As this lens is not located at the stop, additional shift values must be applied to the results obtained using the equations for a lens at the stop.
If the lens were at the stop we would obtain the following values

$S_1=0.007146$ $S_2=-0.009455$
$S_3=0.0075$ $S_4=0.004945$
$S_5=0.0$ $C_1=0.002922$
$C_2=0.0$

For lens 2 the required term in the stop shift equations $E=\bar{h}/h=1$. Using this value we obtain the final contributions from lens 2 as

$S_1=0.007146$ $S_2=-0.002309$
$S_3=-0.004264$ $S_4=0.004945$
$S_5=0.006226$ $C_1=0.002922$
$C_2=0.002922$

The totals for the system are therefore

$S_1=0.023587$ $S_2=-0.000241$
$S_3=0.003236$ $S_4=0.00989$
$S_5=0.006226$ $C_1=0.01461$
$C_2=0.002922$

For this system these values are very close for practical purposes to those obtained from a surface by surface calculation, which includes the effects of lens thickness.

**Interpretation of Seidal Coefficients**

Computation of the Seidal coefficients allows us to evaluate aberrations in a system. Different Seidal coefficients indicate the presence of different aberrations.
A non-zero value will indicate that the following aberrations are present.

$S_1$ Spherical Aberration
$S_2$ Coma
$S_3+S_4$ Field Curvature & Astigmatism
$S_5$ Distortion
$C_1$ or $C_2$ Chromatic Aberration

The following brief discussion on the interpretation of these values should assist in assessing the aberrations of a system.

**Spherical aberration**
The most important aberration for the majority of systems is Spherical Aberration as it occurs over the whole field. The spot size in the paraxial focal plane is given by

$S_1/u_{N+1}$.

It can be reduced by a factor of 4 with a suitable refocus of

$3S_1/8u^2_{N+1}$.

The associated wavefront error is

$S_1/8$;

If $|S_1|$ is less than $7.6\lambda$ the system is likely to be diffraction limited on–axis (if no higher orders are present).

**Coma**
The distance from the chief ray intersection point to the extreme ray in the coma pattern (see page 11) is given by

$3S_2/2u_{N+1}$.

If $|S_2|$ is less than $1.2\lambda$, then the system should be diffraction limited for coma.

**Astigmatism & field curvature**
Off–axis the situation is complicated by the interaction between all the aberrations on the wavefront. For diffraction limited performance look for $|S_3|$ and $|S_4|$ to be lower than $\lambda$. These figures are not absolute values but indicate when a lens will be completely unsuitable. (See Welford for a more complete treatment.)
The astigmatic blur in the paraxial focal plane is an ellipse with dimensions

$(3S_3+S_4)/u_{N+1}$ by $(S_3+S_4)/u_{N+1}$.

**Distortion**
The distortion of the edges of the image as a fraction of the Gaussian image height is given by

$S_5/2H$.

**Chromatic aberration**
If focused in the paraxial focal plane at one end of the spectral range the blur diameter would be $2C_1/u_{N+1}$ due to longituding chromatic aberration.
In simple terms the blur is 1/V times the lens diameter for an infinite object distance.