TWO LENS COMBINATIONS

Two Lens Combinations – Infinite Conjugates

Given two thin lens components with powers K_1 and K_2, separated by a distance d, the power K of the assembly may be calculated using the following equation

$$K = K_1 + K_2 - dK_1K_2$$

The focal length f of the assembly is given by

$$f = \frac{1}{K}$$

The back focal distances f_b measured from vertex V′ of lens 2 is given by

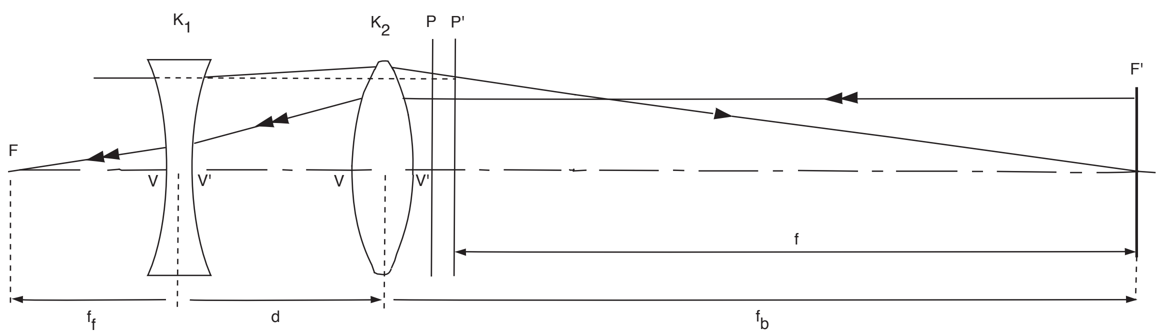
$$f_b = f(1 - dK_1)$$

and the front focal distance f_f measured from vertex V of lens 1 is given by

$$f_f = f(1 - dK_2)$$

Given the focal length and the front and back focal distances the locations of the principal planes P and P' for the assembly can be determined.

If the lenses are thick, the separation d is that between the second principal plane P′ of lens 1 and the first principal plane P of lens 2. Also the the front and back focal distances are measured from the principal plane P of lens 1 and P′ of lens 2 respectively.
(The principal planes for the individual lenses are not shown on the figure.)



Two Lens Combinations – Finite Conjugates

A) If an object distance s, image distance s′, separation d and magnification m are known, then powers K_1 and K_2 of the lenses can be found from the following equations:

$$K_1 = (s - s'/m - d)/sd$$
$$K_2 = (-ms + s' + d)/s'd$$

B) If the focal lengths f_1 and f_2, the magnification m and the total track T are known, then the thin lens separation, object and image distances can be found.

The possible separations of the lenses d are given by the solution of the quadratic equation

$$d^2 - Td + \left[T(f_1 + f_2) + \frac{(m-1)^2 f_1 f_2}{m}\right] = 0$$

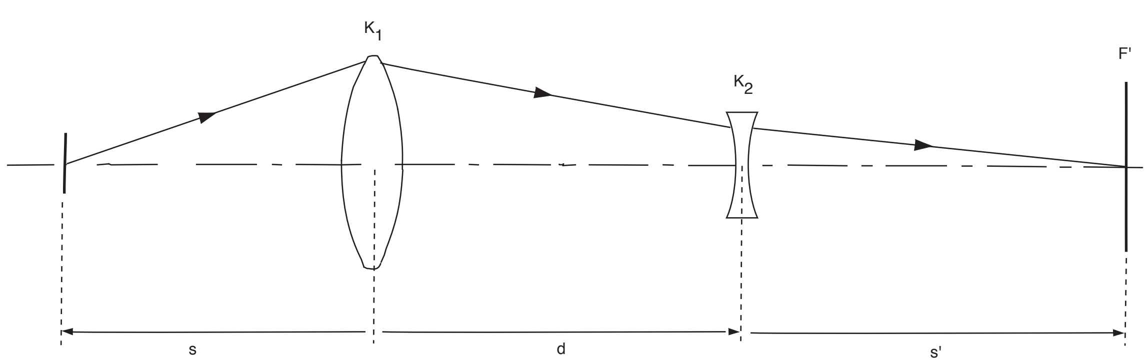
Object distance s is given by

$$s = \frac{(1-m)f_1f_2 + (d-T)f_1}{f_1 + mf_2}$$

and Image distance s' is given by

$$s' = T - d + s$$

[Note. As for the equations for the infinite conjugate case, distances are always referred to the appropriate principal point positions]



Example 3

We use two 75mm focal length Plano–Convex singlets to give a 50mm combined focal length.

Using the formula for the total power of a two lens combination

$$K = K_1 + K_2 - dK_1K_2$$

we require $K=1/50mm^{-1}$, and both K_1 and K_2 are equal to $1/75mm^{-1}$.

Solving for d we find d = 37.5 mm.

As these are thick lenses the separation is between P' of lens 1 and P of lens 2. We have VP=0 for lens 2 and V'P'=-1.7mm for lens 1. The separation between the lenses is therefore 35.8mm.

In addition we can calculate the thin lens back focal length

$f_b = f(1-dK_1)$
$= 50(1-37.5(1/75))$
= 25mm (measured from P' of lens 2).

This gives a physical back focus for the assembly of 23.3mm with these real lenses, taking thickness into account.

The thin lens front focal distance

$f_f = f(1-dK_2)$
= 25mm (measured from P of lens 1).

As VP=0 for lens 1, the actual front focal distance for the combination would also be 25mm.

Example 4

To illustrate the use of the two–lens equations for finite conjugates, suppose that a 10X magnification is required with a total track of 500mm, but with a working clearance of 100mm and a lens separation of around 50mm.

So we have
$m = -10$,
$T = 500$ mm,
$s = -100$
and $d = 50$mm.

For a thin lens system

$T = -s+d+s'$,
giving s'=350 mm.

The relevant equations give

$K_1 = (s-s'/m-d)/sd$
$= \left(-100 - \frac{350}{(-10)} - 50\right)/(-100)50$
$= 0.0230$

$f_1 = 1/K_1$
$= 43.48$ mm

and
$K_2 = (-ms+s'+d)/s'd$
$= (-(-10)(-100)+350+50)/350x50$
$= -0.0343$

$f_2 = 1/K_1$
$= -29.17$ mm

Suppose we choose $f_1 = 50$mm and $f_2 = -30$mm as suitable stock components. Substituting these values into the equation on Theory page 3 results in the quadratic equation

$d^2 - 500d + 28150 = 0$
(with solutions d =64.662 or –109.32mm)

The auxiliary equation given opposite for s, gives

$$s = \frac{(1-(-10))50(-30)+(64.662-500)50}{50+10(-30)}$$
= –109.334 mm, in the first case.

As the first lens is working almost at 1:1 a suitable choice might be an Equi-Convex Lens for which the principal points are separated by

PP'=VV'–VP+V'P'
=4.6 –(1.5)+(–1.5)
=1.6mm.

For the second lens we might choose a Plano–Concave Lens for which

PP'=1.5–0+(–1.0)=0.5mm.

The separation of the principal points of these real components adds up to 2.1mm and this should be subtracted from the total track T when computing d,s and s'. For this particular case we find

d = 64.92mm and
s = –109.0mm.

Remember that d is the separation between P' for lens 1 and P for lens 2, so the actual airgap would be 63.42mm. Also s and s' will be measured from the appropriate principal points P for lens 1 and P' for lens 2.