Theory

GEOMETRICAL IMAGE FORMATION

We will first give a brief introduction to the theory underlying basic geometrical image formation, in order to point out the importance of paraxial optics and aberrations. At the simplest level the paraxial optics approximation determines the location and size of the images, and the magnitude of the aberrations determine the quality of those images.
In almost all cases, the preliminary layout of an optical system is carried out using the paraxial approximation, with refinements in critical areas, based on the level of aberrations, carried out at a later stage.

Geometrical Optics

If we take a light ray passing through a particular sequence of reflecting and refracting surfaces in an optical system, there is in general only one ray linking any two arbitrary points in object space and image space. However, it is possible that certain pairs of points may exist which are linked by an infinite number of different ray paths. In such a case, the two points are called conjugate points, with one being the perfect (or stigmatic) image of the other.

A question which often arises is whether it is possible to devise an optical system which produces a perfect image of an extended three–dimensional volume. It has been proved that were such a perfect system to exist, then the conjugate points would have to be related by a linear relationship. Apart from a few trivial cases (eg. the plane mirror), no such systems are realizable. The failure of a real system to conform to the perfect linear model is caused by its aberrations.

For most real systems, the aberrations are small enough perturbations from the linear model that predictions based on that model are still of practical use, at least in the prelimin–ary stages. Consequently, the linear approximation (known variously as the Gaussian, First Order or Paraxial approximation) is especially important. The term paraxial refers to a region close to the axis of an optical system, for which the sines of angles may be approximated by the angles themselves. This results in a considerable simplification to Snell's law of refraction.

Paraxial Optics

If we confine the discussion to the paraxial region, there are several important parameters which can be defined for an optical system in air.

In the figure on the opposite page, the ray from an object point at infinity would form an image at the back focal point F', at a distance f_b from the rear vertex V'.

If the object were placed at the front focal point F, at distance f_f from the front vertex V, then the image would be formed at infinity.

If, for the case of an image at infinity, lines along the input and output ray directions are extended until they intersect, those intersection points map out the locus of the first principal surface.

The axial separation of the principal surface from its corresponding focal point is the Effective Focal Length (EFL) f.

Applying the same procedure to the case of the object at infinity gives the location of the second principal surface.

In the paraxial approximation these surfaces become planes, which cut the axis at the first and second principal points P and P'.

Knowledge of the principal plane positions of an optical system and its focal length allow the object and image positions and the magnification of the geometrical image to be determined completely, (apart from the effects of aberrations). The equations that must be applied are called the conjugate equations and these are presented in the following column.

Conjugate Equations

In the figure opposite, the object distance s is defined relative to the first principal point P and the image distance s′ relative to the second principal point P'. The sign convention used here is given opposite. In addition the magnification m is defined as negative if the image is inverted with respect to the object.
The object to image distance (total track) is T.
The Gaussian conjugate equations are given here in several forms to allow the most important parameters to be determined from whichever values are known.

$$f = \frac{s\,s'}{s-s'} \qquad s = \frac{f\,s'}{f-s'}$$

$$f = \frac{s'}{1-m} \qquad s = \frac{s'}{m}$$

$$f = \frac{ms}{1-m} \qquad s = \frac{f(1-m)}{m}$$

$$s' = ms \qquad s' = f(1-m)$$

$$s' = \frac{fs}{f+s} \qquad T = \frac{-f(m-1)^2}{m}$$

$$f = \frac{-mT}{(m-1)^2}$$

It is also possible to compute conjugate positions relative to the front and back focal points using the Newton conjugate equations.

$$x = s+f \qquad x' = s'-f$$

$$m = \frac{f}{x} = -\frac{x'}{f} \qquad xx' = -f^2$$