

# PURCHASING INFORMATION

## ENGLISH

IF YOU REQUIRE FURTHER INFORMATION OR ASSISTANCE IN PLACING YOUR ORDER, PLEASE CONTACT THE APPROPRIATE EALING SALES OFFICE. PLEASE SEE PAGES 220-221 FOR THE REPRESENTATIVE IN YOUR REGION.

## FRENCH

Pour toute information supplémentaire, ou si vous avez besoin d'aide pour votre commande, contactez le représentant local d'Ealing dont vous trouverez les coordonnées en pages 220-21.

## GERMAN

Wenn Sie weitere Informationen oder ein Angebot wünschen, wenden Sie sich bitte an Ihr regionales Ealing-Verkaufsbüro. Auf den Seiten 220-221 finden Sie den Repräsentanten in Ihrer Region.

## ITALIAN

Per ulteriori informazioni tecnico-commerciali vi prego di contattare l'ufficio vendite Ealing competente. Nelle pagg. 220-221 troverete i dettagli del rappresentante del vs. paese di appartenenza.

## SPANISH

Si usted requiere de información adicional o apoyo para hacer su pedido, por favor entre en contacto con la oficina de ventas de EALING apropiada. Vea por favor las paginas 220-221 para el representante en su región.

## DUTCH

Als u verdere informatie of hulp bij het plaatsen van uw bestelling vereist, gelieve het locale Ealing verkoopkantoor te contacteren. Zie pagina's 220-221 voor de vertegenwoordiger in uw gebied.

## SWEDISH

För mer information angående produkterna eller för assistans vid beställning, vänligen kontakta anvisat Ealing försäljningskontor. På sidorna 220-221 finner ni representanten i er region.

## KOREAN

제품에 대한 주문 및 상세한 상담이 필요하시면, 한국내의 대리점에 연락하시면 친절히 도와 드릴 것입니다. 대리점 정보를 확인하시려면 페이지 220-221를 확인해 보십시오.

## The Company Behind THE CATALOG

Today Ealing Electro-Optics continues, as it has done for over a hundred years, to serve its customers with high quality optical and mechanical components and instruments.

Ealing was the first to produce a comprehensive catalog of standard components as the R&J Beck Company. With each issue we have increased and refined the content to meet the ever-changing requirements of research and industry. We believe that this catalog is no exception. For most of you it will be an invaluable source of cost-effective, easily-procured quality components.

But Ealing is more than just a catalog. Behind it there lies a wealth of experience and talent able to design, produce, assemble and test components to the highest possible standards.

We value you, our customers- the optical scientists and engineers of the world- and we have devoted many months to ensuring that we have provided you with a catalog that you will want to use time and time again.

Many engineers find our catalog is a handy source of components for incorporation into prototypes. However, a prototype can only provide proof of principle. The final product must be engineered for production and profit. Ealing can help by providing a fully-integrated value-engineered product in which you can have full confidence.



### Technical Engineers

All our applications engineers are fully trained, highly qualified and united in a common goal. They want to understand fully your needs and requirements and to work closely with you to ensure that you are fully satisfied with our service.



### Optical Component Fabrication

Our skilled craftsmen can work most optical materials to very high tolerances and our testing area is fully equipped and controlled to ensure that you receive quality, repeatable components.



### Electronic Assembly

We at Ealing have been incorporating electronic assemblies in our motorized components, instruments and systems for many years.



### Assembly and Test

If your requirement is for a single component, a sub-assembly or a complete instrument, we will exercise the same care to ensure that you have a product that is built to the highest standards.



### Quality Assurance

If you are to depend on our products, it is essential that you have complete confidence in their quality.

  
**Ealing**  
CATALOG, INC.

P.O. Box 1208 5718 Lonetree Blvd. Rocklin CA 95765  
1-800-295-3220

# The History of Ealing Catalog, Inc.

A limited number of firms have the extensive history of Ealing.

1865 - 1903 Publication of R&J Beck Catalog and textbooks



1865- The first catalog was published; "A treatise on the construction, proper use and capabilities of Smith, Beck, and Beck's Achromatic Microscope"

1903- Conrad Beck published "Photographic lenses, a simple treatise" by R. & J. Beck Ltd. London

1950- Reflecting Microscope Objectives were developed on designs from the Wheatstone Laboratory and Medical Research Council Biophysics Research Unit at King's College, London

1920s- The Eros M.T.F. systems were developed

1984- Three-dimensional molecular teaching model, Corey-Pauling (CPK's) was patented by Ealing Corporation

**Ealing**  
CATALOG, INC.

2002- Ealing Catalog, Inc. re-established the Ealing name under private investors

*Future-* Expanding customer service and adding new products while continuing the same high quality you have experienced over the past 100 years.

2007- Expanded distribution for Europe



2008- New Catalog printed and the Beck brand microscopes introduced



1842- Richard Beck produced the world's first microscope with compound achromatic objectives

1848- James Darwin commissioned Smith and Beck to build a microscope, which became a standard product

1847- Richard Beck entered into partnership with Smith to trade as Smith and Beck

1867- The name of the firm was changed to R. & J. Beck

1888- Beck- T.E. Lawrence, better known as 'Lawrence of Arabia', used a plate camera brand Beck- Ensign

1926- The first sound-on-film pictures were made possible by R. & J. Beck's optic design



1900s New optics assemblies designs

1987- Ealing Electro-Optics acquired by 600 Group Ltd., UK

1968- R. & J. Beck Ltd. was acquired by the Ealing Corporation in South Natick, Massachusetts, U.S.A.



1997- Headquarters moved to Northern California having been acquired by Coherent, Inc.

## New Products

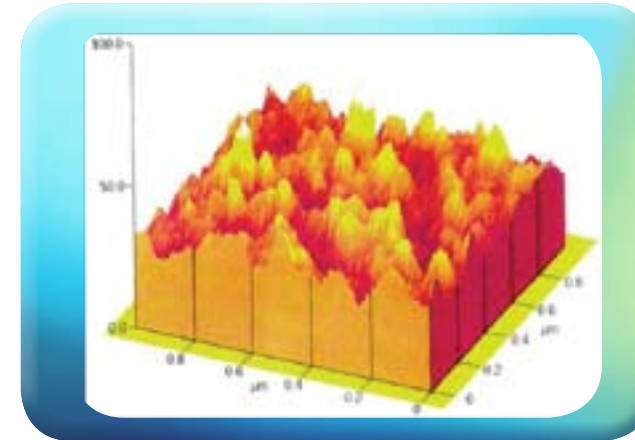


### R&J BECK - IROSCOPE MICROSCOPES

- *Designed for scientific research*
- *Fine focusing mechanism*
- *High intensity halogen illumination*
- *High performance optical elements*
- *Zoom range of 6x - 50x*
- *Full 360° rotating binocular head*
- *Large range of optional accessories for extended capabilities*

*\*See pages 189-200*

## New Products

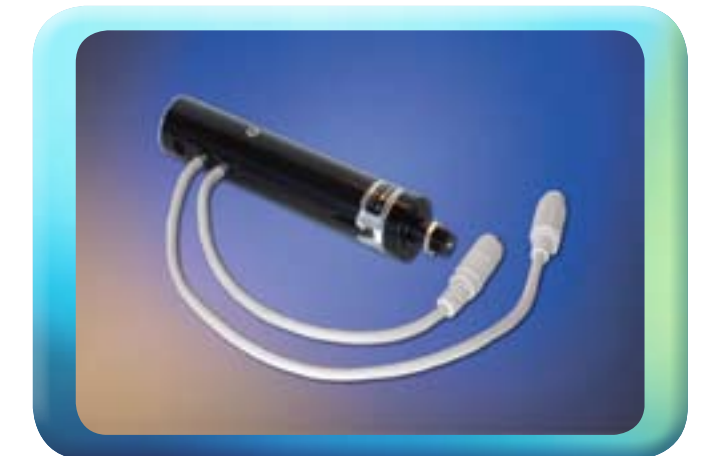


### ADVANCED COATING

- *Ion-assisted e-beam - < 10 Å RMS typical*
  - *APS / Plasma Deposition - < 4 Å RMS typical*
  - *Highly durable, dense, non-shifting*
- \*See pages 50-51*

### ZABER LINEAR ACTUATORS

- *Designed for use with our line of Linear Translation Stages and to replace manual micrometer heads*
  - *Three travel ranges: 13mm, 28mm, and 60mm*
  - *0.1 µm resolution*
  - *Optional knob with center dent to allow for manual positioning*
  - *Models available with vacuum compatibility*
- \*See pages 174-175*



### MICROSCOPE OBJECTIVES

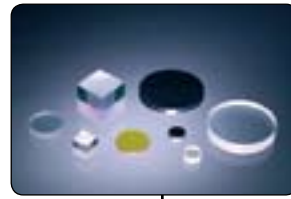
- *Long Working Distances*
- *High Quality Apochromat Design*
- *Ideal for use in Laser Marking and Cutting Semiconductor Circuits*
- *Broadband Spectral Range*

*\*See pages 41-202*



### SINGLE-AXIS CONTROL MODULE

- *Single Axis DC Motor Control*
  - *Standard RS-232/USB PC Interface*
  - *LabView Drivers and demo software*
  - *ROHS Compliant*
- \*See pages 176*



## The Ealing Catalog

◆ <b>Optics</b>	<i>Table of Contents</i>	<b>3</b>
	<i>Tutorial</i>	<b>4-29</b>
	<i>Lenses &amp; Microscope Components</i>	<b>30-43</b>
	<i>Coatings</i>	<b>44-51</b>
	<i>Mirrors, Beamsplitters &amp; Windows</i>	<b>52-63</b>
	<i>Prisms and Polarizers</i>	<b>64-81</b>
	<i>Filters</i>	<b>82-99</b>
	<i>Pinholes</i>	<b>100-105</b>
◆ <b>Opto-Mechanics</b>	<i>Table of Contents</i>	<b>106-107</b>
	<i>Rails</i>	<b>108-111</b>
	<i>Mounting Hardware</i>	<b>112-115</b>
	<i>Mirror and Component Mounts</i>	<b>116-137</b>
	<i>Manual Micropositioners</i>	<b>138-163</b>
	<i>Motorized Micropositioners</i>	<b>164-185</b>
◆ <b>Microscopes &amp; Components</b>	<i>Table of Contents</i>	<b>188-189</b>
	<i>Microscopes</i>	<b>190-200</b>
	<i>Microscope Components</i>	<b>201-205</b>
◆ <b>Index</b>	<i>Product/Subject Index</i>	
	<i>Catalog Number Index</i>	

### Optics

Lenses & Microscope Components

Coatings

Mirrors & Beamsplitters

Prisms & Polarizers

Filters

Pinholes

### Opto-mechanics

Rails

Mounting Hardware

Mirror & Component Mounts

Manual Micro Positioners

Motorized Positioners

### Optical Instruments

Microscopes

Light Sources

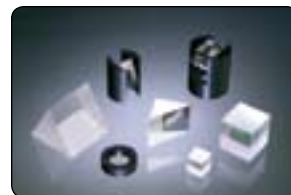
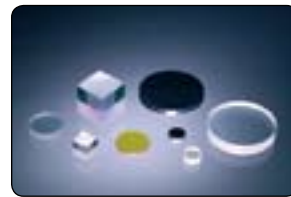
Optics are commonly used in laser systems as well as a wide range of non-laser applications. Selection of the appropriate component is dependent on a number of factors. Laser optics must not only perform the required task but they must withstand illumination with, often high energy, laser radiation. Ealing has extensive experience in the design and manufacture of high quality optical components for use with laser systems.

Shown on the following pages are a wide range of standard optics that will meet many of the more common requirements for laser and photon applications. All of these items are made to the same exacting standards we have demanded for the past 50 years. They have been designed and manufactured to meet the needs of laser research and development engineers and scientists. All items are carried in stock for immediate delivery.

We have one of the largest selections of stock optics available anywhere and you can be confident of the quality you will receive. Optional coatings are also offered and will be applied to our standard uncoated optics on request.

If you need optical components for use in volume manufacturing situations, we will be pleased to provide you with a special quotation. We can design to meet your needs or build to your print.

Whatever your optics requirement, give us a call. Our engineers are ready to answer your questions and help with your selection of stock, OEM, or custom optical components.



## Optics

<b>Tutorial</b>	<b>4</b>
<b>Lenses &amp; Microscope Components</b>	<b>30</b>
<b>Coatings</b>	<b>44</b>
<b>Mirrors &amp; Beamsplitters</b>	<b>52</b>
<b>Prisms &amp; Polarizers</b>	<b>68</b>
<b>Filters</b>	<b>82</b>
<b>Pinholes</b>	<b>100</b>

## Optical Materials

<i>Optical Glass</i>	5-6
<i>Fused Silica and Fused Quartz</i>	7
<i>Calcium Fluoride</i>	8
<i>Germanium</i>	9
<i>Zinc Selenide</i>	10
<i>Sapphire</i>	11
<b>Theory</b>	
<i>Geometrical Image Formation</i>	12
<i>Paraxial Optics</i>	13
<i>Two Lens Combinations</i>	14-15
<i>Paraxial Raytracing</i>	16-19
<i>Aberrations</i>	20-24
<i>Doublets</i>	25-26
<i>Overall System Considerations</i>	27-29

There are a multitude of optical grade glasses available from various manufacturers worldwide. They have a wide variety of optical characteristics which allows aberrations to be corrected to the level required for very demanding imaging applications. Optical glasses are predominantly used from 350nm - 2500nm - in fact within this spectral region there is usually no need to consider any other material. Two of the most important optical characteristics of an optical glass are the refractive index and the dispersion. The dispersion is the index difference between the extreme ends of the spectral region of interest. Another figure sometimes more convenient in discussing the properties of an optical material is the Abbe number, which is related to, and calculated from the dispersion. Although these quantities can be defined using any arbitrary wavelengths, a particular choice which has become the de facto standard in the industry is the use of the three wavelengths d,C,F (587.6, 656.3, 486.1nm respectively). In this case the Abbe number  $V_d$  is defined as

$$V_d = \frac{n_d - 1}{n_F - n_C}$$

Sometimes the refractive index and Abbe number are combined into a six figure glass code, which allows substitute glasses from alternative suppliers to be located with greater ease. This code is formed by using the first three significant figures after the decimal point in the value of  $n_d$  followed by the three most significant digits in the value of  $V_d$ .

Irrespective of the glass manufacturer, it is very common for the Schott designation to be given in the catalog listing of material properties. Most manufacturers produce glasses which are, for practical purposes, identical to many of the more popular glasses in the Schott range, in addition to specialty glasses of their own.

For historical reasons the range of glasses is divided into two subgroups: The crown glasses with  $n_d < 1.6$  and  $V_d > 55$  or  $n_d > 1.6$  and  $V_d > 50$ ; with the remaining glasses known as the flint glasses.

The ideal glass prescription would be a glass with a high value of both  $n_d$  and  $V_d$ , as this would enable components of high refracting power to be constructed from components with shallow curves. In addition the color aberrations and some of the monochromatic aberrations would be greatly reduced. Unfortunately, (in order to increase the refractive index), oxides of heavier elements have to be added to the basic  $\text{SiO}_2$  matrix upon which most optical glasses are made. This generally results in a loss of transmission at the blue end of the spectrum and an increase in the optical dispersion. Glasses with high values of  $n_d$  tend to be flints, and as a general rule an increase in  $n_d$  and  $V_d$  both lead to an increase in material cost (in some cases by an order of magnitude).

By far the most popular optical glass is BK7(517642). It exhibits excellent transmission from 350nm to 2000nm. The transmission tails away slowly in the infrared with a complete extinction beyond 2.7 microns. At the UV end of the spectrum the material is not really usable at wavelengths lower than 300nm.

BK7 is a low index crown glass. It exhibits a good resistance to atmospheric attack, acid and alkali conditions and to staining - all of

these features being desirable both in the final component and during the manufacturing processes that are required. The popularity of the material is such that all the major glass manufacturers produce a functionally identical equivalent, ensuring ready availability. In addition especially large pieces with improved levels of homogeneity and freedom from internal defects can be produced, at additional cost.

The majority of the components for the visible region in the Ealing Catalog are manufactured from BK7, except where special optical or physical properties are required.

Examples of this are in the use of F2(620364) and SF10(728284) glass to give a high spectral separation in some of the dispersing prisms, and LaSFN9(850322) in the micro lenses.

Also a wide variety of crown and flint combinations are used to correct the aberrations in compound lenses, such as the cemented achromatic doublets.

The nominal refractive indices of the four glasses BK7, LaSFN9, F2 and SF10 at any wavelength in the region 0.365 to 1.060 microns can be computed using the following expression:

$$n^2 = A_0 + A_1 \lambda^2 + A_2 \lambda^{-2} + A_3 \lambda^{-4} + A_4 \lambda^{-6} + A_5 \lambda^{-8}$$

where  $\lambda$  is the wavelength in microns and the coefficients for each glass are as shown below.

Glass Type	BK7	LaSFN9	F2	SF10
Coefficient				
$A_0$	2.2718929	3.2994326	2.5554063	2.8784725
$A_1$	-1.0108077x10 <sup>-2</sup>	-1.1680436x10 <sup>-2</sup>	-8.746150x10 <sup>-3</sup>	-1.0565453x10 <sup>-2</sup>
$A_2$	1.0592509x10 <sup>-2</sup>	4.0133103x10 <sup>-2</sup>	2.2494787x10 <sup>-2</sup>	3.3279420x10 <sup>-2</sup>
$A_3$	2.0816965x10 <sup>-4</sup>	1.3263988x10 <sup>-3</sup>	8.6924972x10 <sup>-4</sup>	2.0551378x10 <sup>-3</sup>
$A_4$	-7.6472538x10 <sup>-6</sup>	4.7438783x10 <sup>-6</sup>	-2.4011704x10 <sup>-5</sup>	-1.1396226x10 <sup>-4</sup>
$A_5$	4.9240991x10 <sup>-7</sup>	7.8507188x10 <sup>-6</sup>	4.5365169x10 <sup>-6</sup>	1.6340021x10 <sup>-5</sup>

Further data and transmission curves are shown on the next Page.

# Optical Materials

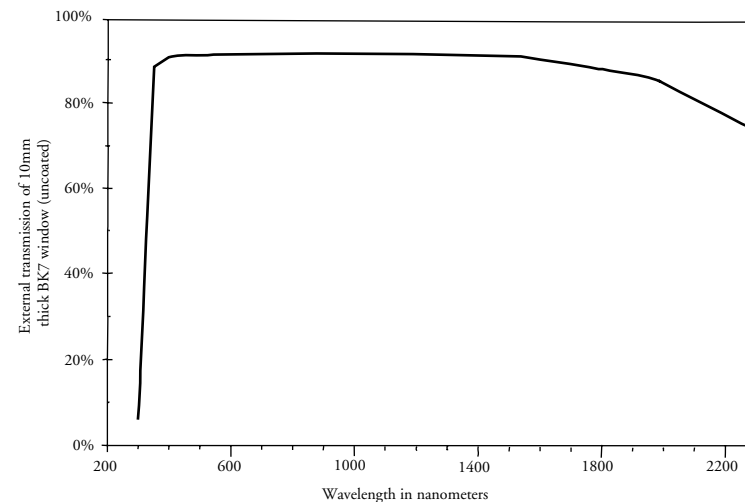
## Optical Glass

Other important data and the transmission curves for these materials are given below.

Material	BK7 (517642)	LaSFN9 (850322)	F2(620364)	SF10(728284)
Abbe Number $V_d$	64.17	32.17	36.37	28.41
Density (g/cm <sup>3</sup> )	2.51	4.44	3.61	4.28
Coefficient of Expansion (10 <sup>-6</sup> /K) (Room Temperature)	7.1	7.4	8.2	7.5
Specific Heat (J/g/K)	0.858	-	0.557	0.465
Thermal Conductivity (W/m/K)	1.114	-	0.780	0.741
Youngs Modulus (10 <sup>9</sup> N/mm <sup>2</sup> )	81	109	58	64
Poissons Ratio	0.208	0.286	0.225	0.232
Knoop Hardness	520	510	370	370

Refractive index versus  $\lambda$

$\lambda$ (nm)	Material			
	BK7	LaSFN9	F2	SF10
365.0	1.53626	-	1.66621	-
404.7	1.53024	1.89844	1.65063	1.77578
435.8	1.52669	1.88467	1.64202	1.76197
486.1	1.52238	1.86899	1.63208	1.74648
546.1	1.51872	1.85651	1.62408	1.73430
587.6	1.51680	1.85026	1.62004	1.72825
632.8	1.51509	1.84489	1.61656	1.72309
656.3	1.51432	1.84526	1.61503	1.72085
852.1	1.50981	1.82997	1.60672	1.70889
1060.0	1.50669	1.82293	1.60191	1.70229



# Optical Materials

## Fused Silica and Fused Quartz

### Fused Silica and Fused Quartz

Both of these materials exhibit transmission characteristics which bear some similarity to the optical glasses with which they are closely related. However, their simpler chemical composition, in particular the absence of some of the heavier metal oxides, leads them to have improved transmission in the Ultraviolet region of the spectrum. Special grades of material may be specified which are pre-treated to reduce the water content. This eliminates characteristic O - H absorption lines in the near Infrared region of the spectrum although at the expense of transmission in the Ultraviolet. The spectral transmission curves are given below.

The physical properties of Fused Quartz and Fused Silica are similar, and in applications where the materials are chosen for their strength, chemical inertness or their resistance to elevated temperature or shock, the less expensive Fused Quartz may be the preferred option.

Refractive index versus  $\lambda$

$\lambda$ (nm)	Index
214.4	1.53372
248.3	1.50840
265.2	1.50000
302.2	1.48719
334.1	1.47976
365.0	1.47454
404.7	1.47962
435.8	1.46669
486.1	1.46313
546.1	1.46008
587.6	1.45846
656.3	1.45637
852.1	1.45247
1014.0	1.45024
1530.0	1.44427
1970.0	1.43852
2325.4	1.43293

The fundamental difference between Fused Quartz and Fused Silica is in the method of manufacture. Most Fused Quartz is produced by the melting and re-fusing of silica sand and natural quartz. This means that trace impurities in the raw materials used can be carried across into the final material matrix. The presence of heavy metallic ions is in part responsible for absorption at Ultraviolet wavelengths or for unacceptable levels of absorption in high power visible lasers.

By comparison Fused Silica is produced by the flame hydrolysis of a silica halide. The resultant material is hence much purer, and free from the sites which could cause absorption in the Ultraviolet or problems with internal absorption in the use of high power lasers.

Either of these materials is the most suitable choice for use in high power lamp sources. As they are far more resistant to thermal shock than optical glass, the final choice would be dependent on the spectral range required.

The refractive index of Fused Silica and Fused Quartz at various wavelengths may be calculated using the following Sellmeier type formula with the coefficients due to Malitson. (J.Opt.Soc.Am. 55p1205 (1965))

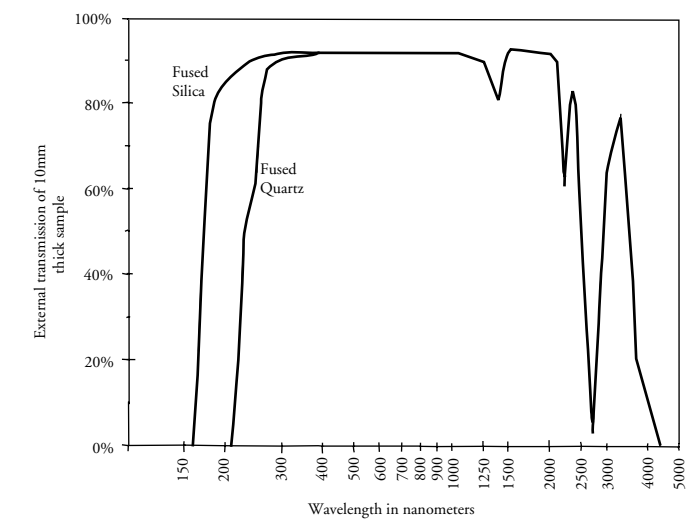
$$n^2 = 1 + \frac{A_0 \lambda^2}{\lambda^2 - B_0} + \frac{A_1 \lambda^2}{\lambda^2 - B_1} + \frac{A_2 \lambda^2}{\lambda^2 - B_2}$$

where

$$\begin{aligned} A_0 &= 0.6961663 \\ B_0 &= 0.004679148 \\ A_1 &= 0.4079426 \\ B_1 &= 0.01351206 \\ A_2 &= 0.8974794 \\ B_2 &= 97.934003 \end{aligned}$$

and  $\lambda$  is the wavelength in microns.

Abbe Number $V_d$ :	67.8
Density:	2.203 g/cm <sup>3</sup>
Softening Point:	1730 °C
Annealing Point:	1180 °C
Specific Heat:	0.76 J/g/K at 20°C
Thermal Conductivity:	1.39 W/m/K
Thermal Expansion:	0.55x10 <sup>-6</sup> /°C
Youngs Modulus:	72.8 x 10 <sup>9</sup> N/mm <sup>2</sup>
Hardness (Mohs):	5-7



# Optical Materials

## Calcium Fluoride

### Calcium Fluoride

Calcium Fluoride (CaF<sub>2</sub>) has a useful transmission over the spectral range from 0.2 - 8.0 microns. The low refractive index (1.35 - 1.51) means that it may be used without recourse to an anti - reflection coating.

It may be used as an alternative to Fused Silica in the Ultraviolet region, especially as a crown type material for assemblies which require reduced levels of chromatic aberration. In addition, the high Abbe number in the visible region, and the anomalous dispersion characteristics, render it useful for correcting visible systems for secondary color aberration (for example, apochromatic microscope objectives).

It is the least expensive material, with visible transmission, that also covers the complete 3 - 5 micron band.

Calcium Fluoride is slightly soluble in water, although the surfaces should be expected to withstand several years exposure to normal atmospheric conditions. Calcium Fluoride is also susceptible to thermal shock, reducing its usefulness somewhat with high radiance sources.

The refractive index of Calcium Fluoride at various wavelengths may be calculated using the following Sellmeier type formula with the coefficients due to Malitson. (Appl.Opt. 2,p1103(1963)).

$$n^2 = 1 + \frac{A_0 \lambda^2}{\lambda^2 - B_0} + \frac{A_1 \lambda^2}{\lambda^2 - B_1} + \frac{A_2 \lambda^2}{\lambda^2 - B_2}$$

where

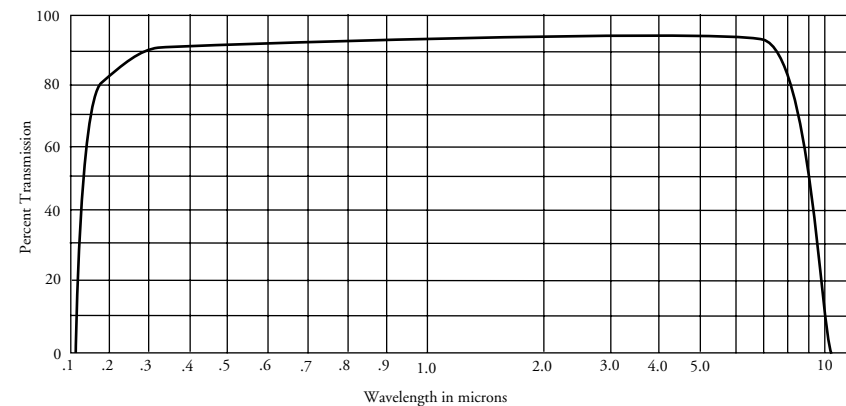
$$\begin{aligned} A_0 &= 0.5675888 \\ B_0 &= 0.00252643 \\ A_1 &= 0.4710914 \\ B_1 &= 0.01007833 \\ A_2 &= 3.8484723 \\ B_2 &= 1200.5560 \end{aligned}$$

and  $\lambda$  is the wavelength in microns

Abbe Number $V_d$ :	94.9
$V_{3-5}$ :	21.7
Density:	3.179 g/cm <sup>3</sup>
Melting Point:	1360°C
Specific Heat:	0.857 J/g/K at 0°C
Thermal Conductivity:	1.037W/m/K at 0°C
Thermal Expansion:	18.9x10 <sup>-6</sup> /K at 20°C
Youngs Modulus:	76x10 <sup>3</sup> N/mm <sup>2</sup>
Solubility:	0.00177 g/100 g H <sub>2</sub> O (at 26°C)

Refractive index versus  $\lambda$

$\lambda$ (nm)	Index
248.3	1.46793
265.2	1.46233
334.1	1.44852
365.0	1.44490
404.7	1.44451
435.8	1.43949
486.1	1.43703
546.1	1.43494
587.6	1.43388
656.3	1.43246
852.1	1.43002
1014.0	1.42879
1530.0	1.42642
1970.0	1.42401
2325.4	1.42212
3507.0	1.41398
5018.8	1.39873
7464.4	1.36070



# Optical Materials

## Germanium

### Germanium

Germanium is a semiconductor material, which has high internal transmittance for radiation in the wavelength range from 1.8 - 12 microns. It is particularly favored for use in the 3 - 5 and 8 - 12 micron regions, for which the atmosphere is transparent. In addition the lower cost of the raw material compared to the possible alternatives is a strong factor in its favor. Its high refractive index (approx 4.0) allows components of high refractive power to be constructed without excessive thickness or extreme curvatures. This keeps material usage down and reduces the processing time in quantity production. The high index also ensures an exceptional single wavelength performance for a "best form" singlet constructed from Germanium. The extremely high Abbe number in the 8- 12 micron band, and the relatively high value in the 3 - 5 micron band, means that in most cases special achromatizing measures are not required. A drawback of the high refractive index is the high uncoated surface reflection losses of 36% per surface, giving an uncoated component an

external transmittance of 40% . This makes the application of an AR coating essential in virtually all practical situations. Fortunately the high refractive index can actually simplify the coating design in some cases, and a range of coatings is available to meet the majority of requirements.

The optimum form of material for use in most optical applications is n - type doped material with a resistivity in the range of 5 - 40  $\Omega$ cm. This material, which is doped with a controlled amount of Antimony(Sb), exhibits a lower absorption loss than other forms of the material, provided the temperature of the material is kept below 50 °C. For high power applications, especially those which involve CW lasers or lasers with high energy pulses and a fast repetition rate, it is preferable to use Zinc Selenide. The problem with Germanium in these applications is its susceptibility to thermal runaway. If energy is absorbed in the component the local temperature near the beam may increase because the thermal conductivity of Germanium is too low for the heat to be dissipated. If this happens then the local value of the absorption coefficient will also rise. There is the possibility of a continual and uncontrolled rise in temperature leading to component failure. Even

where water cooling is used, the thermal gradient required to allow the heat away may only reach a steady state condition at a point where thermal distortion of the beam has upset the system performance.

The refractive index of Germanium at various wavelengths can be calculated using the following formula due to Salzberg and Villa:- (J.Opt.Soc.Am. 43 p 579 (1953))

$$n = A + BL + CL^2 + D \lambda^2 + E \lambda^4$$

where

A =	3.99931
B =	0.391707
C =	0.163492
D =	- 0.0000060
E =	0.000000053

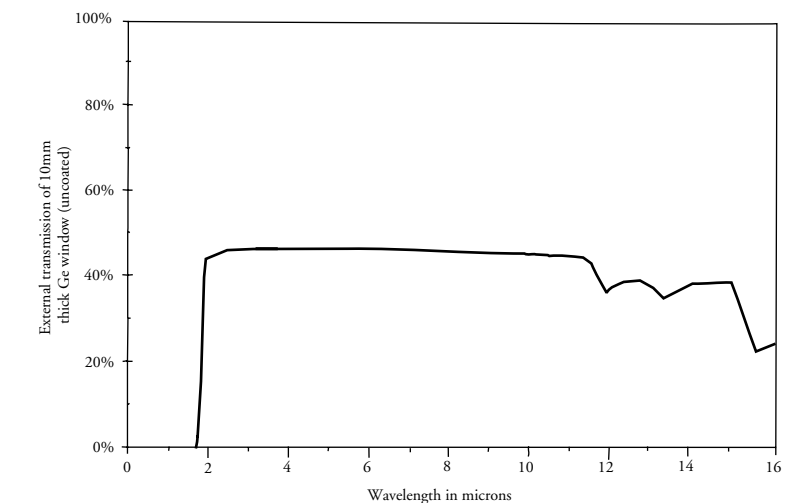
$$L = \frac{1}{(\lambda^2 - 0.028)}$$

and  $\lambda$  is the wavelength in microns

Abbe Number $V_{3-5}$ :	101.5
$V_{8-12}$ :	988
Density:	5.327 g/cm <sup>3</sup>
Melting Point:	936 °C
Specific Heat:	0.32J/g/K at 0°C
Thermal Conductivity:	59.9W/m/K at 20°C
Thermal Expansion:	5.75x10 <sup>-6</sup> /K at 20 °C
Youngs Modulus:	103x10 <sup>3</sup> Nmm <sup>-2</sup>

Refractive index versus  $\lambda$

$\lambda$ (microns)	Index
2.0	4.10827
2.5	4.06645
3.0	4.04495
3.5	4.03239
4.0	4.02439
4.5	4.01898
5.0	4.01514
5.5	4.01232
6.0	4.01018
6.5	4.00852
7.0	4.00721
7.5	4.00616
8.0	4.00531
8.5	4.00461
9.0	4.00403
9.5	4.00356
10.0	4.00317
10.5	4.00286
11.0	4.00261
12.0	4.00227
13.0	4.00213
14.0	4.00217



# Optical Materials

## Zinc Selenide

### Zinc Selenide

Zinc Selenide is a transparent polycrystalline material with a transmission range from 0.5 - 15 microns. The material is produced by a CVD (Chemical Vapor Deposition) process, which ensures that the extrinsic impurity levels are kept low. The purity of the material minimizes the bulk absorption loss figures.

Although the material is more expensive than Germanium, the ability to use visual alignment aids (such as a HeNe laser), can reduce the costs incurred by the end user. This is especially so if many optical components are used in sequence.

The absorption losses are not only lower than Germanium, they are also far more stable with temperature. This ensures that the risk of thermal runaway leading to component failure is reduced, and this material should always be considered in any high power laser application. In addition the possible dangers of

Refractive index versus  $\lambda$

$\lambda$ (microns)	Index
1.0	2.48882
1.5	2.45708
2.0	2.44620
2.5	2.44087
3.0	2.43758
3.5	2.43517
4.0	2.43316
4.5	2.43132
5.0	2.42953
5.5	2.42772
6.0	2.42584
6.5	2.42388
7.0	2.42181
7.5	2.41961
8.0	2.41728
8.5	2.41481
9.0	2.41218
9.5	2.40939
10.0	2.40644
10.5	2.40331
11.0	2.40000
12.0	2.39281
13.0	2.38481
14.0	2.37593

having an unpredictable path through misaligned optics is far less because of the visual checks which can be carried out prior to firing up the laser.

The average refractive index value of approximately 2.4 gives uncoated reflection losses of 17% per surface, and an external transmittance of 69% in the principal beam. Usually the main issue is not the loss of radiation so much as the unwanted stray light problem. Some form of AR coating is recommended on ZnSe components and coatings are available to meet both narrow band and wide band requirements.

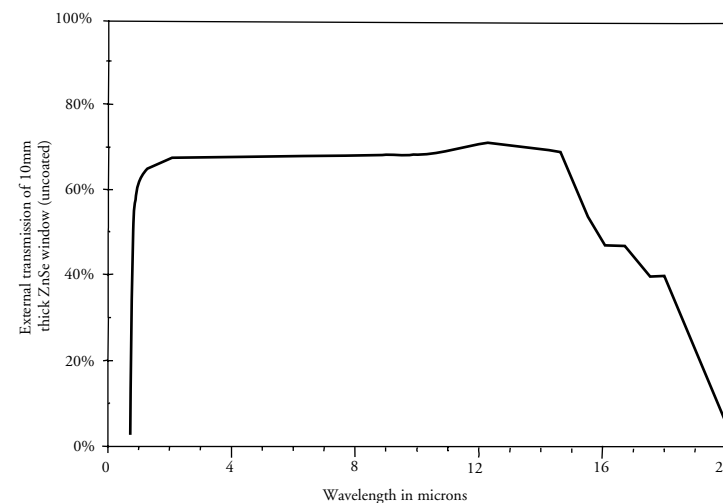
The refractive index of Zinc Selenide at various wavelengths may be calculated using the following Sellmeier type formula with the coefficients due to Feldman, Horowitz, Waxler and Dodge. "Optical Materials Characterization Final Report February 1, 1978 - September 30, 1978", NBS Technical Note 993.

$$n^2 = 1 + \frac{A_0 \lambda^2}{\lambda^2 - B_0} + \frac{A_1 \lambda^2}{\lambda^2 - B_1} + \frac{A_2 \lambda^2}{\lambda^2 - B_2}$$

where  $A_0=4.2980149$   
 $B_0=0.036888196$   
 $A_1=0.62776557$   
 $B_1=0.14347626$   
 $A_2=2.8955633$   
 $B_2=2208.4920$

and  $\lambda$  is the wavelength in microns.

Abbe Number  $V_{d,3-5}$ : 178  
 $V_{d,8-12}$ : 57.5  
 Density: 5.27 g/cm<sup>3</sup>  
 Specific Heat: 0.34 J/g/K at 25 °C  
 Thermal Conductivity: 18 W/m/K at 25 °C  
 Thermal Expansion: 7.1x10<sup>-6</sup>/K at 0 °C  
 Youngs Modulus: 67x10<sup>3</sup>N/Kmm<sup>2</sup>



# Optical Materials

## Sapphire

### Sapphire

This material has excellent mechanical and optical properties. It is extremely hard (9 on the mohs scale) and has a strength which is retained even at elevated temperatures. In addition it is resistant to virtually all forms of chemical attack. It has a transmission band which extends from the UV at around 0.18nm, throughout the visible and near IR regions to 5.5 microns.

The high cost of this versatile material naturally tends to limit its use to the following areas:

- 1) As a durable single lens in detector applications which do not require multi - component correction.
- 2) As a protective window to shield more delicate optics from harsh environments such as debris from laser cutting processes. The window can frequently be kept extremely thin, saving on material cost and lowering any possible internal absorption. Vigorous cleaning methods can also be used with less

Refractive index versus  $\lambda$

$\lambda$ (nm)	Index
265.2	1.83365
334.1	1.80181
365.0	1.79358
404.7	1.78582
435.8	1.78120
486.1	1.77558
546.1	1.77077
587.6	1.76823
656.3	1.76494
852.1	1.75887
1014.0	1.75546
1530.0	1.74659
1970.0	1.73835
2325.4	1.73055
3507.0	1.69501
5145.6	1.61510

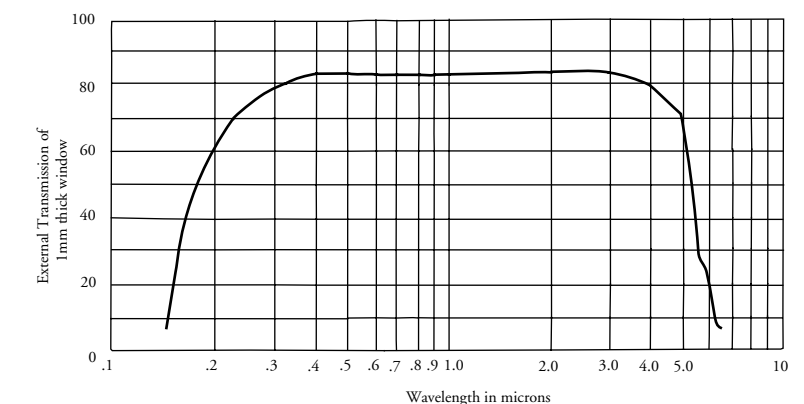
danger of scratching the window and causing losses due to surface scatter.

3) It has also been used more recently in the form of spherical balls in fiber coupling applications.

Depending on the wavelength at which it is used the Fresnel reflection losses vary from 5% - 20% per surface. However at the wavelengths where these losses are highest the refractive index of 1.7 - 1.8 is almost ideal for the application of a single layer coating of MgF<sub>2</sub>.

The internal structure of the material is anisotropic, which means that certain properties have a dependence on the direction in which they are measured in the bulk material. The most significant of these properties is the birefringence introduced, which could be a problem in a particularly demanding application. Fortunately the thinness of the substrates used is usually a sufficient protection against this source of image degradation.

The refractive index of Sapphire at various wavelengths may be



calculated using the following Sellmeier type formula with the coefficients due to Malitson. (J.Opt.Soc.Am. 52p 1377 (1962))

$$n^2 = 1 + \frac{A_0 \lambda^2}{\lambda^2 - B_0} + \frac{A_1 \lambda^2}{\lambda^2 - B_1} + \frac{A_2 \lambda^2}{\lambda^2 - B_2}$$

where  $A_0=1.023798$   
 $B_0=0.00377588$   
 $A_1=1.058264$   
 $B_1=0.0122544$   
 $A_2=5.280792$   
 $B_2=321.3616$

and  $\lambda$  is the wavelength in microns

Abbe Number  $V_d$ : 72.2  
 Density: 3.98 g/cm<sup>3</sup>  
 Melting Point: 2055 °C  
 Specific Heat: 756 J/kg/°C at 18 °C  
 Thermal Conductivity: 24 W/m/K  
 Thermal Expansion: 6.5x10<sup>-6</sup>/K  
 Youngs Modulus: 345x10<sup>3</sup>N/mm<sup>2</sup>  
 (Properties are dependent on orientation)

# Theory

## Geometrical Image Formation

We will first give a brief introduction to the theory underlying basic geometrical image formation, in order to point out the importance of paraxial optics and aberrations. At the simplest level the paraxial optics approximation determines the location and size of the images, and the magnitude of the aberrations determine the quality of those images.

In almost all cases, the preliminary layout of an optical system is carried out using the paraxial approximation, with refinements in critical areas, based on the level of aberrations, carried out at a later stage.

### Geometrical Optics

If we take a light ray passing through a particular sequence of reflecting and refracting surfaces in an optical system, there is in general only one ray linking any two arbitrary points in object space and image space. However, it is possible that certain pairs of points may exist which are linked by an infinite number of different ray paths. In such a case, the two points are called conjugate points, with one being the perfect (or stigmatic) image of the other. A question which often arises is whether it is possible to devise an optical system which produces a perfect image of an extended three-dimensional volume. It has been proved that were such a perfect system to exist, then the conjugate points would have to be related by a linear relationship. Apart from a few trivial cases (eg. the plane mirror), no such systems are realizable. The failure of a real system to conform to the perfect linear model is caused by its aberrations.

For most real systems, the aberrations are small enough perturbations from the linear model that predictions based on that model are still of practical use, at least in the preliminary stages. Consequently, the linear approximation (known variously as the Gaussian, First Order or Paraxial approximation) is especially important. The term paraxial refers to

a region close to the axis of an optical system, for which the sines of angles may be approximated by the angles themselves. This results in a considerable simplification to Snell's law of refraction.

### Paraxial Optics

If we confine the discussion to the paraxial region, there are several important parameters which can be defined for an optical system in air.

In the figure on the opposite page, the ray from an object point at infinity would form an image at the back focal point  $F'$ , at a distance  $f_b$  from the rear vertex  $V'$ .

If the object were placed at the front focal point  $F$ , at distance  $f_f$  from the front vertex  $V$ , then the image would be formed at infinity.

If, for the case of an image at infinity, lines along the input and output ray directions are extended until they intersect, those intersection points map out the locus of the first principal surface.

The axial separation of the principal surface from its corresponding focal point is the Effective Focal Length (EFL)  $f$ .

Applying the same procedure to the case of the object at infinity gives the location of the second principal surface.

In the paraxial approximation these surfaces become planes, which cut the axis at the first and second principal points  $P$  and  $P'$ .

Knowledge of the principal plane positions of an optical system and its focal length allow the object and image positions and the magnification of the geometrical image to be determined completely, (apart from the effects of aberrations). The equations that must be applied are called the conjugate equations and these are presented in the following column.

### Conjugate Equations

In the figure opposite, the object distance  $s$  is defined relative to the first principal point  $P$  and the image distance  $s'$  relative to the second principal point  $P'$ . The sign convention used here is given opposite. In addition the magnification  $m$  is defined as negative if the image is inverted with respect to the object. The object to image distance (total track) is  $T$ .

The Gaussian conjugate equations are given here in several forms to allow the most important parameters to be determined from whichever values are known.

$$f = \frac{s s'}{s - s'} \quad s = \frac{f s'}{f - s'}$$

$$f = \frac{s'}{1 - m} \quad s = \frac{s'}{m}$$

$$f = \frac{ms}{1 - m} \quad s = \frac{f(1 - m)}{m}$$

$$s' = ms \quad s' = f(1 - m)$$

$$s' = \frac{fs}{f + s} \quad T = \frac{-f(m - 1)^2}{m}$$

$$f = \frac{-mT}{(m - 1)^2}$$

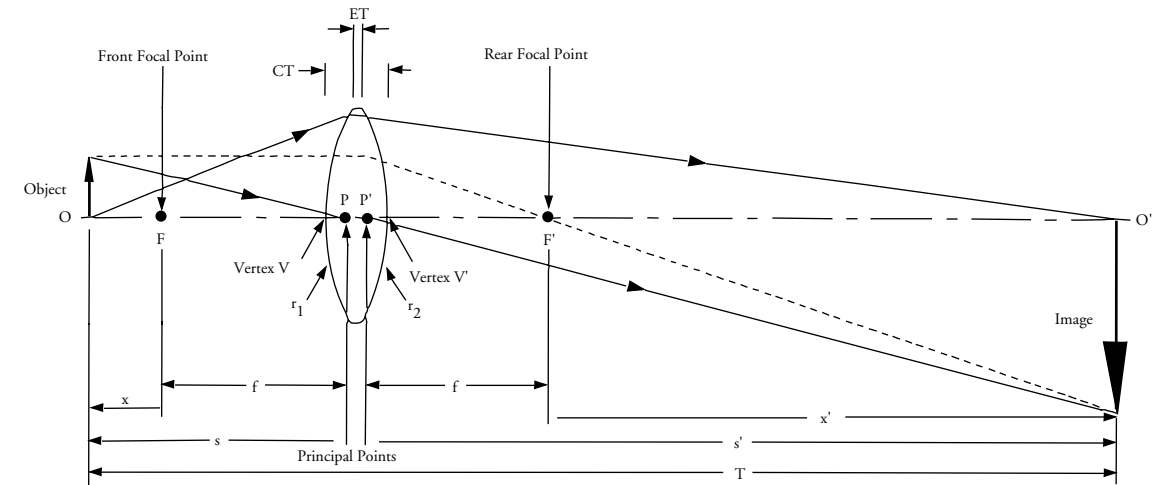
It is also possible to compute conjugate positions relative to the front and back focal points using the Newton conjugate equations.

$$x = s + f \quad x' = s' - f$$

$$m = \frac{f}{x} = -\frac{x'}{f} \quad xx' = -f^2$$

# Theory

## Paraxial Optics



The convention used here is:

$s$  +ve if direction  $PO$  is to the right  
 $s'$  +ve if direction  $P'O'$  is to the right  
 $x$  +ve if direction  $FO$  is to the right  
 $x'$  +ve if direction  $F'O'$  is to the right  
 $m$  -ve if image is inverted  
 radii +ve if center lies to the right of surface

The focal length  $f$  of a thick lens may be calculated using the following formula

$$\frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n - 1)^2 CT}{nr_1 r_2}$$

The principal plane positions may then be found from

$$VP = \frac{-CT(n-1)f}{nr_2} \quad \text{or} \quad V'P' = \frac{-CT(n-1)f}{nr_1}$$

The magnitude of sag drop on the surface is given by

$$\text{Sag} = \left\{ |R| - \sqrt{R^2 - D^2/4} \right\}$$

where  $R$  is the radius of the curvature of that surface and  $D$  the diameter of the component.

Many expressions in optics are simplified by considering the power  $K$ , given by

$$K = 1/f$$

A common unit of lens power is the diopter which is the power of a lens with a focal length of 1 meter.

In many cases, the center thickness  $CT$  produces a second order change to the overall focal length. For a change in wavelength, the focal length  $f'_\lambda$  at a new wavelength  $\lambda'$  can be calculated from the initial focal length  $f_\lambda$  at wavelength  $\lambda$  by the following equation

$$f'_\lambda = f_\lambda \left( \frac{n_\lambda - 1}{n_{\lambda'} - 1} \right)$$

### Example 1

To determine the magnification and image position of the 43 - 0058 BK7 Plano - Convex Lens for an object located 200mm from the curved face.

The first principal point  $P$  for this lens is located at the curved surface as  $VP = 0$ .

Taking into account the sign convention  $s = -200$  mm. The nominal focal length  $f = 40$ mm.

Using the equations and the information given in the previous table

$$s' = fs/(f+s) = 40 \cdot (-200)/(40 + (-200)) = 50 \text{ mm (measured from } P')$$

Given  $V'P' = -1.6$ mm the paraxial image is formed 48.4 mm from the plano face of the lens.

$$\text{Magnification } m = s'/s = 50/(-200) = -0.25 \text{ (the image is inverted)}$$

### Example 2

Solving the same problem using the Newton conjugate equations

$$x = s + f = -200 + 40 = -160 \text{ mm}$$

$$x' = -f^2/x = -(40)^2/(-160) = 10 \text{ mm}$$

$$m = f/x = 40/(-160) = -0.25$$

$$s' = x' + f = 10 + 40 = 50 \text{ mm}$$

# Theory

## Two Lens Combinations

### Two Lens Combinations- Infinite Conjugates

Given two thin lens components with powers  $K_1$  and  $K_2$ , separated by a distance  $d$ , the power  $K$  of the assembly may be calculated using the following equation

$$K = K_1 + K_2 - dK_1K_2$$

The focal length  $f$  of the assembly is given by

$$f = \frac{1}{K}$$

The back focal distances  $f_b$  measured from vertex  $V'$  of lens 2 is given by

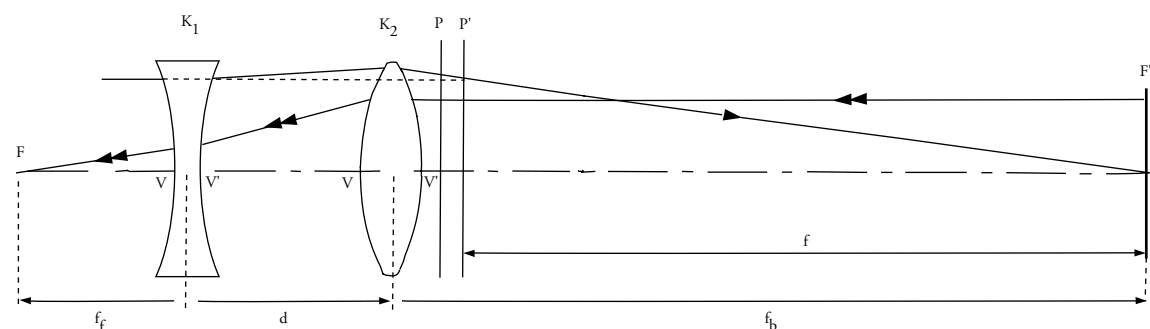
$$f_b = f(1 - dK_1)$$

and the front focal distance  $f_f$  measured from vertex  $V$  of lens 1 is given by

$$f_f = f(1 - dK_2)$$

Given the focal length and the front and back focal distances the locations of the principal planes  $P$  and  $P'$  for the assembly can be determined.

If the lenses are thick, the separation  $d$  is that between the second principal plane  $P'$  of lens 1 and the first principal plane  $P$  of lens 2. Also the the front and back focal distances are measured from the principal plane  $P$  of lens 1 and  $P'$  of lens 2 respectively. (The principal planes for the individual lenses are not shown on the figure.)



### Two Lens Combinations - Finite Conjugates

A) If an object distance  $s$ , image distance  $s'$ , separation  $d$  and magnification  $m$  are known, then powers  $K_1$  and  $K_2$  of the lenses can be found from the following equations:

$$K_1 = (s - s'/m - d)/sd$$

$$K_2 = (-ms + s' + d)/s'd$$

B) If the focal lengths  $f_1$  and  $f_2$ , the magnification  $m$  and the total track  $T$  are known, then the thin lens separation, object and image distances can be found.

The possible separations of the lenses  $d$  are given by the solution of the quadratic equation

$$d^2 - Td + [T(f_1 + f_2) + \frac{(m-1)^2 f_1 f_2}{m}] = 0$$

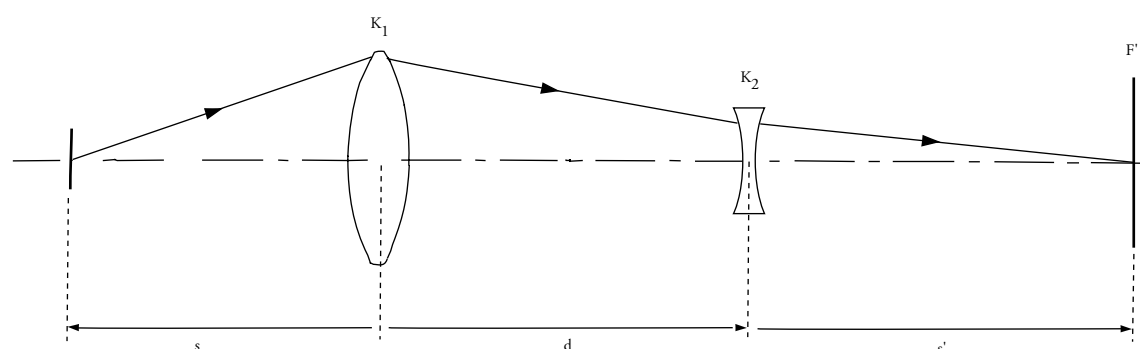
Object distance  $s$  is given by

$$s = \frac{(1-m)f_1 f_2 + (d-T)f_1}{f_1 + m f_2}$$

and Image distance  $s'$  is given by

$$s' = T - d + s$$

[ Note. As for the equations for the infinite conjugate case, distances are always referred to the appropriate principal point positions]



# Theory

## Two Lens Combinations

### Example 3

To use two 75mm focal length Plano - Convex singlets 43 - 0173, (Page 199), to give a 50mm combined focal length.

Using the formula for the total power of a two lens combination

$$K = K_1 + K_2 - dK_1K_2$$

we require  $K=1/50\text{mm}^{-1}$ , and both  $K_1$  and  $K_2$  are equal to  $1/75\text{mm}^{-1}$ .

Solving for  $d$  we find  $d = 37.5\text{mm}$ .

As these are thick lenses the separation is between  $P'$  of lens 1 and  $P$  of lens 2. In the table on page 199,  $VP=0$  for lens 2 and  $V'P' = -1.7\text{mm}$  for lens 1. The separation between the lenses is therefore 35.8mm.

In addition we can calculate the thin lens back focal length

$$f_b = f(1 - dK_2)$$

$$= 50(1 - 37.5(1/75))$$

$$= 25\text{mm (measured from } P' \text{ of lens 2).}$$

This gives a physical back focus for the assembly of 23.3mm with these real lenses, taking thickness into account.

The thin lens front focal distance

$$f_f = f(1 - dK_1)$$

$$= 25\text{mm (measured from } P \text{ of lens 1).}$$

As  $VP=0$  for lens 1, the actual front focal distance for the combination would also be 25mm.

### Example 4

To illustrate the use of the two - lens equations for finite conjugates, suppose that a 10X magnification is required with a total track of 500mm, but with a working clearance of 100mm and a lens separation of around 50mm.

So we have

$$m = -10,$$

$$T = 500\text{mm},$$

$$s = -100$$

and  $d = 50\text{mm}$ .

For a thin lens system

$$T = -s + d + s',$$

giving  $s' = 350\text{mm}$ .

The relevant equations give

$$K_1 = (s - s'/m - d)/sd$$

$$= (-100 - 350(-10))/(-100)50$$

$$= 0.0230$$

$$f_1 = 1/K_1$$

$$= 43.48\text{mm}$$

and

$$K_2 = (-ms + s' + d)/s'd$$

$$= (-(-10)(-100) + 350 + 50)/350 \times 50$$

$$= -0.0343$$

$$f_2 = 1/K_2$$

$$= -29.17\text{mm}$$

Suppose we choose  $f_1 = 50\text{mm}$  and  $f_2 = -30\text{mm}$  as suitable stock components. Substituting these values into the equation on the opposite page results in the quadratic equation

$$d^2 - 500d + 28150 = 0$$

(with solutions  $d = 64.662$  or  $-109.32\text{mm}$ )

The auxiliary equation given opposite for  $s$ , gives

$$s = \frac{(1 - (-10))50(-30) + (64.662 - 500)50}{50 + 10(-30)}$$

$$= -109.334\text{mm, in the first case.}$$

As the first lens is working almost at 1:1 a suitable choice might be a 43 - 1023 Equi - Convex Lens (Page 201) for which the principal points are separated by

$$PP' = VV' - VP + V'P'$$

$$= 4.6 - (1.5) + (-1.5)$$

$$= 1.6\text{mm.}$$

For the second lens we might choose a Plano - Concave Lens 43 - 1577, (Page 202), for which

$$PP' = 1.5 - 0 + (-1.0) = 0.5\text{mm.}$$

The separation of the principal points of these real components adds up to 2.1mm and this should be subtracted from the total track  $T$  when computing  $d, s$  and  $s'$ . For this particular case we find

$$d = 64.92\text{mm and}$$

$$s = -109.0\text{mm.}$$

Remember that  $d$  is the separation between  $P'$  for lens 1 and  $P$  for lens 2, so the actual airgap would be 63.42mm. Also  $s$  and  $s'$  will be measured from the appropriate principal points  $P$  for lens 1 and  $P'$  for lens 2.

# Theory

## Paraxial Raytracing

Paraxial raytracing through multi-lens systems consisting of thin lenses

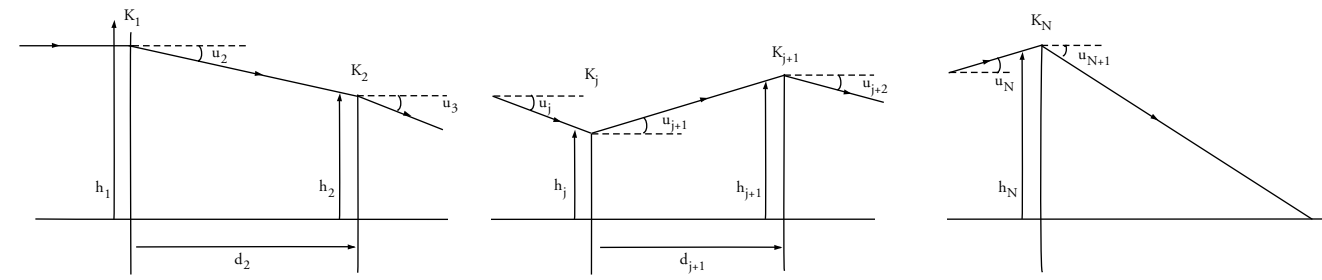
The paraxial raytrace equations are an invaluable aid in determining many useful pieces of information about an optical system, including the following

- 1) Approximate ray heights at each lens. These are required to determine the clear diameters for each lens to avoid unacceptable light loss.
- 2) The focal lengths and positions of the principal planes of systems consisting of two or more elements.
- 3) The object and image positions, and the magnification of multi-element systems.

For further applications of paraxial raytracing, and for a more thorough introduction to the techniques, you may wish to consult one of the following books, which also discuss the tracing of rays outside the paraxial region

“Modern Optical Engineering” by W.J. Smith  
 “Elements of Optical System Design” by D.O’Shea

Both these books are available from Ealing and are listed in the Textbooks Section 5.



When using paraxial raytracing to examine an optical system, the 4 types of ray outlined here can often provide particularly useful information.

The parameters required are shown in the figure below. Here we use the following sign convention

Slope angles  $u$  are defined to be +ve if the ray slopes upwards to the right. Ray heights  $h$  are defined to be +ve if the ray is above the axis.

The parameters required are the separations  $d_j$  between  $j$ th and  $(j+1)$ th component and the powers  $K_j$  of the components.

[For thick lenses the values  $d_j$  are separations between the second principal plane of the  $j$ th component and the first principal plane of the  $(j+1)$ th component.]

Successive application of the following equations, traces the path of the paraxial ray through the optical system.

$$\begin{aligned} u_{j+1} &= u_j - h_j K_j \\ h_{j+1} &= h_j + u_{j+1} d_{j+1} \end{aligned}$$

for  $j=1 \dots N$  where  $N$  is the number of lenses.

**Ray 1** A ray with  $u_1 = 0$  and  $h_1 = 1$ . Tracing this ray allows the calculation of the focal length  $f$  and back focal distance  $f_b$ , from

$$f = -\frac{1}{u_{N+1}} \quad \text{and} \quad f_b = -\frac{h_N}{u_{N+1}}$$

### Reverse Raytracing

To trace rays in reverse, a simple manipulation of the last equations gives the procedure:

$$\begin{aligned} h_j &= h_{j+1} - u_{j+1} d_{j+1} \\ u_j &= u_{j+1} + h_j K_j \end{aligned}$$

starting with  $u_{N+1}$  and  $h_N$  and working through until  $j=1$

**Ray 2** A ray with  $u_{N+1} = 0$  and  $h_N = 1$ . Tracing this ray in reverse allows a check of the focal length  $f$  to be made and also gives the front focal distance  $f_f$ , from

$$f = \frac{1}{u_1} \quad \text{and} \quad f_f = \frac{h_1}{u_1}$$

# Theory

## Paraxial Raytracing

**Ray 3** A ray traced for infinite object distance from the object position with  $u_1$  set to some arbitrary value and  $h_1 = -s u_1$ , where  $s$  is the object distance introduced earlier on Page 13. This ray allows the location of the image to be found, relative to the last lens, from

$$s' = -\frac{h_N}{u_{N+1}}$$

To obtain useful information about the ray heights and angles of the paraxial marginal ray, take either Ray 1 or Ray 3 for the case of an infinitely distant object or a finite object distance respectively. Then scale all  $h_j$  and  $u_j$  values so that the value of  $h$  at the stop is equal to the stop radius, or  $|u_{N+1}|$  is equal to the output numerical aperture. The magnification of the  $j$ th component is given by

$$m = \frac{u_j}{u_{j+1}}$$

**Ray 4** This ray should pass through the center of the aperture stop (the quantities have a 'bar' placed over them by convention) so that  $\bar{h}=0$  at that surface. Either a forwards or backwards trace maybe carried out with an arbitrary value of  $\bar{u}$  at the stop.

For an infinite object distance scale the  $\bar{u}$  and  $\bar{h}$  values to make the initial field angle  $\bar{u}$  correct.

For a finite object distance, determine the current object height of the arbitrary ray from  $\bar{h}_j + s \bar{u}_j$  and then scale all  $h$  and  $u$  values until the object height is the desired value. Alternatively, one can work with the image height  $\bar{h}_N + s' \bar{u}_{N+1}$ , if that quantity has a target value.

A particularly useful feature of the linear nature of the paraxial equations is the ability to combine the paths of any two known paraxial rays to determine the path of another paraxial ray. Two uses are given here.

Use of the paraxial raytrace to determine clear radii.

To determine the required clear radii of components, the rays passing through the upper and lower edges of the aperture stop should be traced. In the absence of vignetting, the heights of these rays at surface  $j$  are given by  $(\bar{h}_j + \bar{h}_j)$  and  $(\bar{h}_j - \bar{h}_j)$  respectively. The required clear radius is the larger of these two in magnitude, that is  $|\bar{h}_j| + |\bar{h}_j|$ .

**Use of the Paraxial raytrace to determine the range of angles present in radiation.**

The range of angles in any space can be found by forming the sum  $|\bar{u}_j| + |\bar{u}_{j+1}|$ . This can be useful in determining the optimum location to place components which have a strong angle sensitive behavior, such as Interference Filters and Polarizers.

### Lagrange Invariant H

If we form the product

$$H = n u \eta$$

where  $n$  is the refractive index and  $u$  the paraxial marginal angle in object space and  $h$  is the object height, then we find that this is equal to the corresponding product  $n' u' \eta'$  formed from the parameters in image space.

The value  $H$  is known as the Lagrange invariant and has important consequences in many areas of optics.

When the object is at infinity, this form of the equation above becomes indeterminate in object space and is replaced by the alternative form

$$H = n h \theta$$

where  $h$  is the paraxial marginal ray height and  $\theta$  is the paraxial chief ray angle in radians.

Additionally,  $H$  may be found in any space of an optical system given the refractive index  $n$ , and the paraxial marginal and chief ray parameters  $u$ ,  $h$ ,  $\bar{u}$  and  $\bar{h}$  from

$$H = n(u\bar{h} - \bar{u}h)$$

One implication of the Lagrange invariant is in illumination calculations especially the concentration of light onto fibers.

For example, given a collection cone angle and image patch size, the value of  $H$  is defined and no single channel optical system can improve collection efficiencies beyond a certain level. Increasing the solid angle of the radiation from the source automatically reduces the area of the source seen by the same amount. Of course, if the source is highly directional in output there may be changes in the total energy flux, but there will be a point of diminishing returns.

# Theory

## Paraxial Raytracing

### Example 5

The system used in Example 3 (previous page) can also be treated using the paraxial raytracing equations.

If we set an initial beam diameter of 15mm, this is a paraxial ray height  $h_1$  of 7.5mm; also for an object at infinity  $u_1=0$ .

Following the procedure of the paraxial raytrace we obtain

$$\begin{aligned} u_2 &= u_1 - h_1 K_1 \\ &= 0 - (7.5)(1/75) \\ &= -0.1 \\ h_2 &= h_1 + d_2 u_2 \\ &= 7.5 + 37.5(-0.1) \\ &= 3.75 \\ u_3 &= u_2 - h_2 K_2 \\ &= -0.1 - 3.75(1/75) \\ &= -0.15 \end{aligned}$$

The focal length is given by

$$\begin{aligned} f &= -h_1/u_3 \\ &= -7.5/(-0.15) = 50 \text{ mm} \end{aligned}$$

and the back focus by

$$\begin{aligned} f_b &= -h_2/u_3 \\ &= -3.75/(-0.15) = 25 \text{ mm.} \end{aligned}$$

Once again the necessary adjustments must be made if the lenses are of finite thickness, by referring all distances to the appropriate principal points.

If we place the stop at the first lens with a field angle of 0.1 radians, then the values for the paraxial chief ray are as follows

$$\begin{aligned} u_1 &= 0.1 \\ h_1 &= 0 \\ u_2 &= 0.1 \\ h_2 &= 3.75 \\ u_3 &= 0.05 \end{aligned}$$

The image height  $h$  is given by

$$h_2 + f_b u_3 = 5 \text{ mm}$$

which is also equal to  $f\theta$ .

### Paraxial Raytracing and Gaussian Beams

Another useful application is in determining the beam waist position and radius of a Gaussian beam. To do this, trace the following two rays from the input beam waist location, at distance  $d_1$  in front of the input lens.

$$\begin{aligned} \text{Ray 1 - } & h_0 = \omega_0 \text{ (the } 1/e^2 \text{ radius of the beam at the input waist),} \\ & u_1 = 0 \\ \text{Ray 2 - } & h_0 = 0, \\ & u_1 = \lambda/\pi\omega_0^2 \text{ (where } \lambda \text{ is the wavelength). This is the far field semi-divergence angle.} \end{aligned}$$

At any position, the beam diameter is given by

$$(h^2 + \bar{h}^2)^{1/2}$$

The distance of output beam waist relative to the appropriate principal plane of lens  $j$  can be found from

$$\frac{-(h_1 u_{j+1} + \bar{h}_1 \bar{u}_{j+1})}{u_{j+1}^2 + \bar{u}_{j+1}^2}$$

The radius of that waist is given by

$$\omega_j = \frac{\lambda}{p(u_{j+1}^2 + \bar{u}_{j+1}^2)^{1/2}}$$

and the far field semi angle of divergence by

$$\theta_j = (u_{j+1}^2 + \bar{u}_{j+1}^2)^{1/2}$$

### Example 6

To compute the position and radius of the beam waist produced by a 50mm focal length lens, when used with a Helium-Neon laser with a beam waist diameter of 1.2mm located 25mm in front of the first principal plane of the lens.

In this case

$$\begin{aligned} \text{Ray 1 - } & h_0 = 0.6 \text{ mm,} \\ & u_1 = 0 \\ \text{Ray 2 - } & h_0 = 0 \text{ mm} \\ & u_1 = \frac{0.6328 \times 10^{-3}}{\pi \cdot 0.6} \\ & = 3.357 \times 10^{-4} \text{ rads.} \\ & \text{(the farfield semi-divergence angle)} \end{aligned}$$

At the lens, using  $h_1 = h_0 + d_1 u_1$

$$\begin{aligned} h_1 &= 0.6 \\ \bar{h}_1 &= (3.357 \times 10^{-4}) \cdot 25 \\ &= 8.393 \times 10^{-3} \text{ mm} \end{aligned}$$

and following refraction, using

$$\begin{aligned} u_2 &= u_1 - h_1 K_1 \\ \bar{u}_2 &= -0.012 \text{ rads.} \\ \bar{u}_2 &= 1.678 \times 10^{-4} \text{ rads.} \end{aligned}$$

Using the appropriate equations in the previous column, we find the output waist is located 49.98mm from the second principal plane of the lens with a beam waist radius of 16.78  $\mu\text{m}$ .

For this particular case, beam waist location is not greatly shifted from the paraxial back focus. However this is not always the case with Gaussian beams, especially where the initial divergence is large.

# Theory

## Paraxial Raytracing

### Diffraction Effects

Before we discuss the aberrations of an optical system, it is necessary to point out that even for a perfect system there is still a fundamental limit to the resolving power or spot size due to the influence of diffraction.

The ray theory of light is an approximation to the true physical situation. More accurately light is made up of electromagnetic waves, which can exhibit effects such as interference and diffraction. The rays, however, still have an important role in the theory as they indicate the

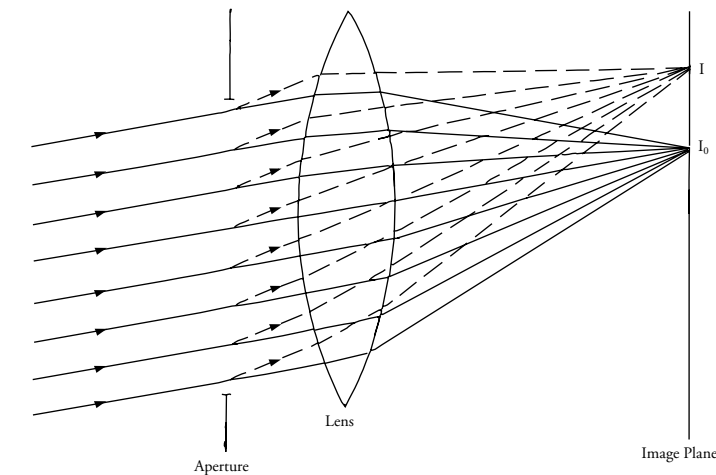
normal to the wavefront.

The figure shows the rays associated with a plane wavefront arriving at an aperture in close proximity to a lens. The aperture modifies the wavefront emerging on the output side. It becomes a combination of the undisturbed portion plus additional components which are scattered by the rim of the aperture. The net effect is to produce a wavefront which is effectively the superposition of an infinite number of plane wavefronts, each having an amplitude which depends on its direction.

The action of the lens is to focus each of these plane wavefronts at a

different lateral location in the back focal plane of the lens. The undiffracted component is the strongest in the example shown and focuses at position  $I_0$ . One of the diffracted plane waves is shown focusing at position I.

The geometrically predicted infinitely small spot focus at position  $I_0$  becomes a finite sized spot with a complicated variation of intensity located near to, though not necessarily centered about,  $I_0$ . The intensity at any point depends on the strength of the component plane wave which is focused there.



### Airy Pattern

For an incident wavefront from an axial object position, arriving at a well centered lens with a circular aperture, the image takes on the familiar Airy pattern shown in the figure. The oscillatory nature of the intensity arises due to scattered contributions from the edges of the aperture moving progressively in and out of phase as the radial distance from the geometrically predicted image is increased.

The form of the Airy pattern is given by

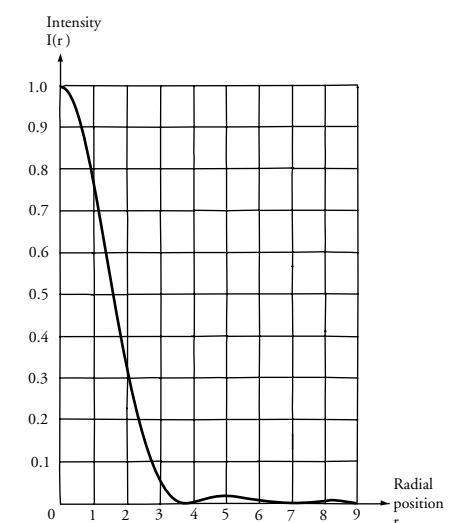
$$I(\rho) = \left( \frac{2J_1(\rho)}{\rho} \right)^2$$

where  $J_1(\rho)$  is a Bessel Function of the first kind. The first zero of  $I(\rho)$  occurs at  $\rho=3.833$ . The quantity is related to the actual radial distance  $r$  from the center of the geometrical spot by

$$\rho = \frac{2\pi NA r}{\lambda}$$

where  $\lambda$  is the wavelength, and NA is the numerical aperture.

The radius to the first dark ring is therefore  $\frac{0.61 \lambda}{NA}$



# Theory

## Aberrations

### Chromatic Aberration

The presence of material dispersion causes the refractive index to vary with wavelength. This means that strictly speaking the results of the paraxial approximation are only true for the central design wavelength.

A useful parameter in assessing the magnitude of chromatic aberration is the Abbe number (V- value). This is defined in terms of the refractive indices  $n_1, n_2$ , and  $n_3$  at three wavelengths  $\lambda_1, \lambda_2$  and  $\lambda_3$  (the design wavelength and the long and short wavelength limits of the spectral band respectively) by

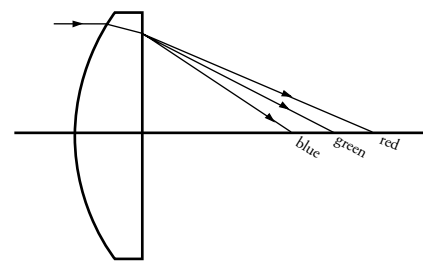
$$V = \frac{n_1 - 1}{n_3 - n_2}$$

The first two chromatic errors to appear are a variation with wavelength of the paraxial image plane position and image height. These are known respectively as longitudinal and lateral chromatic aberration. The accompanying figures show the behavior of a typical lens.

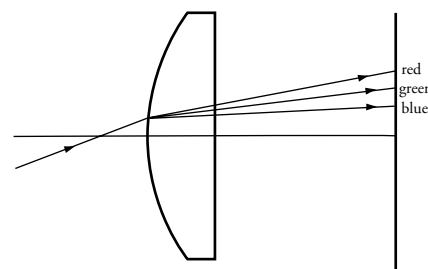
The fractional change in focal length over the spectral range of interest is given by  $1/V$ . So for BK7 lenses used between the wavelengths 486.1nm and 656.6nm where the value is  $V_d = 64.17$ , the change in focal length is approximately 1.6 per cent.

If the chief ray passes centrally through the lens (i.e. the lens is at the stop) then it is undeflected and the contribution to lateral chromatic aberration is zero. It is worth pointing out that lateral chromatic aberration is a shift in image height with the image plane fixed at the position given for the design wavelength.

Changes of lens shape have no influence on the chromatic aberration to a first order approximation, and at least two components of different materials are required for its correction. There are exceptions such as the two lens Dyalte design and binary lenses which combine refracting and diffracting power, but these are beyond the level of discussion here.



Longitudinal Chromatic Aberration



Lateral Chromatic Aberration

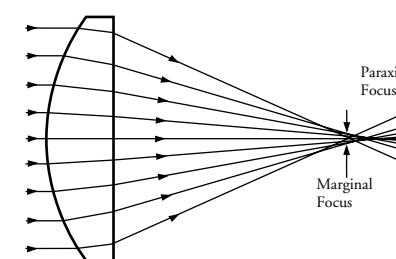
# Theory

## Aberrations

### Seidel Aberrations

The most important aberrations in the majority of applications are the Seidel (also known as Primary or Third order) aberrations. These are the first aberrations to have an impact on the image quality as the aperture and field angles are increased beyond the point at which the paraxial approximation ceases to remain accurate. There are five Seidel monochromatic aberrations. In addition there are two Primary chromatic aberrations. These are the most significant in the majority of applications.

#### Primary Spherical Aberration



The figure shows the situation for an infinitely distant object conjugate. As the incident ray height at the lens is increased, the angle of incidence also increases. The deflection of the ray at that boundary increases more rapidly than the amount predicted by the paraxial approximation.

The net effect is for a lens with spherical surfaces to exhibit excess power for rays farther from the optical axis, bringing these rays to a focus closer to the lens. The difference between the paraxial and marginal focus positions is called the longitudinal spherical aberration.

A more convenient measure of the magnitude of the aberration in most cases is to examine the distance from the axis at which rays pierce a particular focal plane. This image criterion is the transverse spherical aberration. In the Seidel approximation, the ray error is proportional to the 3rd power of the initial ray height.

A smaller spot size can be obtained by selecting an appropriate focal plane. In the geometrical optics approximation (i.e. ignoring diffraction effects) and in the absence of higher order aberrations, the optimum plane lies 3/4 of the way from the paraxial focus to the marginal focus. This results in a spot size which is 1/4 of the size which would be obtained at the paraxial focus position.

If the system is diffraction limited then the wavefront is a more reliable indicator of image quality. The wavefront error associated with primary spherical aberration is dependent on the 4th power of the aperture. The optimum focal plane in this situation would be midway between the paraxial and marginal focal positions. This location maximizes the intensity at the center of the diffraction spot.

A reduction in the amount of spherical aberration can be achieved in several ways. The simplest method is to reduce the aperture, and here the improvement can be quite dramatic. For example, a reduction of 20% in lens aperture halves the geometrical blur size and reduces the wavefront error to 40% of its initial value. Another technique of reducing the spherical aberration is to change the shape of the lens, a technique known as "bending." The spherical aberration of a lens is usually reduced if the deflections of the ray which occur at the two surfaces are made more nearly equal. Depending on the conjugates, i.e. the magnification, there is an optimum shape and orientation for the lens. By extending this technique to the use of more than one lens in a system, it can be seen that sharing the power between the surfaces of several lenses results in a reduction of the magnitude of the spherical aberration.

With glass of refractive index close to 1.5 the best positive lens form for use with infinite conjugates is a bi-convex lens with a 6:1 ratio in its radii

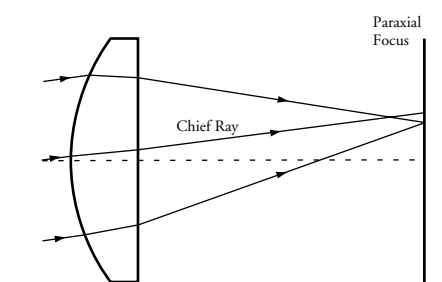
of curvature, with the steepest side positioned towards the infinite conjugate. However the performance gain over a plano-convex lens is minimal and for reasons of cost the plano-convex form is nearly always used for this application.

For 1:1 imaging, the equi-convex lens is the favourable shape as it produces an almost equal angular deflection of the ray path per surface.

The final technique for the reduction in spherical aberration involves introducing lenses of opposite power, which have controlled amounts of spherical aberration of opposing signs. An example of such a case is the cemented achromatic doublets covered in a later section.

In complex lens systems, one or more of these techniques may be applied to correct the overall lens assembly. In addition surfaces with non-spherical form, or materials with controlled departures of the refractive index from the homogeneous case, may be utilized.

#### Primary Coma



Coma is the first of the lens aberrations to appear as the conjugate points are moved away from the optical axis. In the figure, a parallel input beam is shown approaching a plano-convex lens at an oblique angle. The ray at the upper edge of the lens has a higher angle of incidence with the curved surface than the ray at the lower edge. By analogy with the case of spherical aberration described previously, the deflection of the upper ray will be greater, and it will intersect the chief ray closer to the lens than the ray from the lower edge.

Optics

Lenses & Microscope Components

Coatings

Mirrors & Beamsplitters

Prisms & Polarizers

Filters

Pinholes

Opto-mechanics

Rails

Mounting Hardware

Mirror & Component Mounts

Manual Micro Positioners

Motorized Positioners

Optical Instruments

Microscopes

Light Sources

Optics

Lenses & Microscope Components

Coatings

Mirrors & Beamsplitters

Prisms & Polarizers

Filters

Pinholes

Opto-mechanics

Rails

Mounting Hardware

Mirror & Component Mounts

Manual Micro Positioners

Motorized Positioners

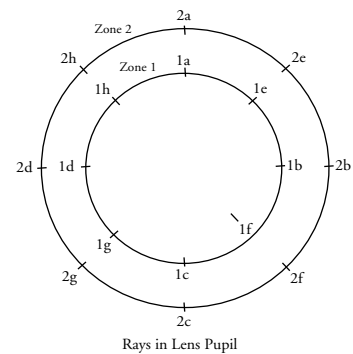
Optical Instruments

Microscopes

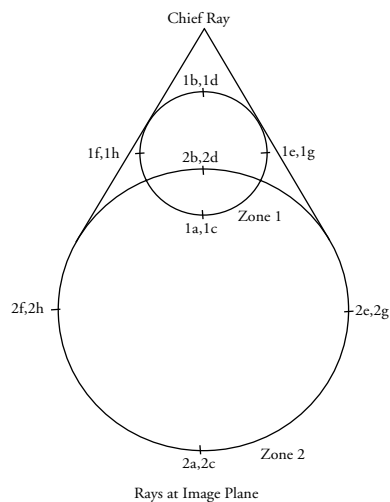
Light Sources

# Theory

## Aberrations



Rays in Lens Pupil



Rays at Image Plane

As shown in the figure above, rays passing through any circular zone of the lens pass through the paraxial image plane in a circular pattern. The center of this pattern experiences an increasing lateral shift away from the point of intersection of the principal ray with the paraxial image plane. In addition the radius of each circle increases as the selected zone at the lens is increased in diameter. Input rays at diametrically opposite points in the lens pupil fall onto the same point in the paraxial image plane.

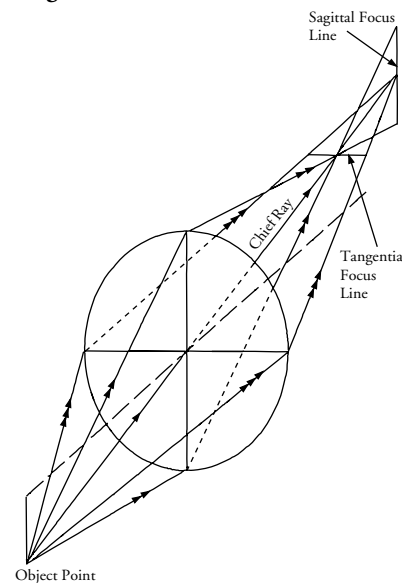
It is the comet-like appearance of the image associated with this aberration that gives rise to the name Coma. The magnitude of the primary coma can be reduced by stopping down the lens or by choosing an appropriate bending for the lens in a similar way to the case of spherical aberration. However, for primary coma, the transverse aberration and wavefront error vary as the 2nd and 3rd powers of the ray height respectively, so the

gain is not so dramatic. The optimum shape of the lens to reduce coma is such as to produce a pseudo-symmetry to the incident beam and is quite close to that which minimizes spherical aberration.

An additional method of reducing coma is to move the stop, so that the off-axis beam translates laterally across the lens and takes up a position where the deflections at the top and bottom are approximately equal. Equations given in a later section will indicate that the reduction of the coma is only possible where there is residual spherical aberration in the system.

It is worth commenting that complex lens arrangements, which exhibit a degree of symmetry about the stop, are usually substantially free from coma.

### Astigmatism and Field Curvature



Astigmatism and Field Curvature are the aberrations from which a lens suffers if it is used off-axis at a low aperture. For single thin lenses the magnitude of these aberrations is proportional to the lens power.

Two principal sections occur - the tangential (or meridional) plane, which contains the object point and the optical axis, and the sagittal (or radial) plane which passes through

the object point and is perpendicular to the tangential plane. For the simple case shown in the figure, the rays in the tangential plane focus closer to the lens than those in the sagittal plane, the discrepancy in the focus position being known as astigmatism.

In the absence of other types of aberration, the sagittal and tangential foci are line foci, while at other planes the shape of the beam is of an elliptical cross-section. Midway between the two line foci lies the meridial surface, where the spot shape is circular. This surface is the optimum choice for many applications.

The shift in focus position is proportional to the square of the image height for primary astigmatism, the positions of the S and T foci mapping out two paraboloidal surfaces as the field position is varied.

Astigmatism can be reduced by varying the stop position, provided that spherical aberration or coma is non-zero. For a lens corrected for all three image defects a multi-component system is almost always required, the shape of the components being in most cases critical and different from those of standard catalog lenses.

Field curvature also results in the foci of off-axis points falling on a paraboloidal surface known as the Petzval surface. It is not possible to remove this aberration by choosing a different stop position or by changing the bending of a single lens. In some cases a compromise may be found if astigmatism of opposing sign is introduced.

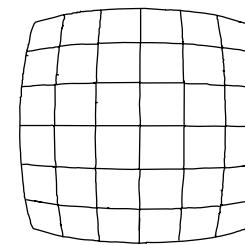
Adjusting the detector position by scanning or by curving a photo-sensitive material can be used in some cases, as field curvature by itself does not degrade the imagery at any individual field point.

In general the field curvature can only be reduced by using several lenses of opposing powers. This is usually only beneficial if a complex lens is used which is also well corrected for astigmatism.

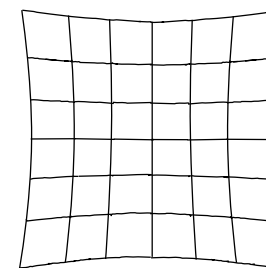
# Theory

## Aberrations

### Distortion



Barrel Distortion



Pincushion Distortion

Distortion is produced when the chief rays intersect the image surface at heights different from those predicted by the paraxial approximation. The cases of pincushion and barrel distortion are shown in the figure above. The dependence of distortion on stop position is strong. A thin lens placed at the stop will exhibit no distortion, and this is also the case for a lens arrangement which is symmetrical about the stop.

In most applications involving simple catalog components, the degradation of the image quality due to astigmatism and field curvature will have reached unacceptable levels before distortion becomes a problem.

Primary distortion is proportional to the 3rd power of the field height. In certain applications it may be compensated for by scanning the image by means of micro-positioning equipment or by pre-distortion of the object, such as might be carried out on a CRT using appropriate deflection voltages.

### Computing Approximate Seidel Aberrations

It is sometimes useful to be able to get an approximate value for the magnitude of the aberrations without using real (finite) raytracing equations.

When using simple components the Seidel approximation is often sufficiently accurate. Here, at least, if higher order aberrations are present, they are usually accompanied by Seidel aberrations of a high magnitude. For this reason expressions are given for the evaluation of the Seidel aberration coefficients of thin lenses.

Most simple catalog components, can be considered as thin lenses for the purpose of evaluating the aberrations. If they are thick relative to their focal length, they are likely to be operating at a fast F/No. In this case the higher order aberrations need to be considered as well.

Even in cases where the Seidel approximation is not accurate, their computation can identify situations where the lens choice is obviously inadequate for the task and point out potential problem areas in an imaging system.

The equations which follow are in a similar form to those in "Aberrations of Optical Systems" by W.T.Welford (Adam Hilger 1986). This book is available from Ealing, (see Section 5). For an interpretation of the Seidel coefficients, see the next Page.

### Equations for a thin lens at the stop

To compute the Seidel coefficients for a thin lens it is necessary to know the following parameters

- 1)  $a_0, a_1, a_2, b_0$  and  $b_1$  which may be found listed in the text of each lens type. The values are sufficiently accurate for work in the region of the design wavelength.
- 2) The height of the paraxial ray  $h$ , for the on-axis object point at the lens in question.
- 3) The Lagrange Invariant  $H$ .
- 4) The value of the Abbe Number  $V$  and the refractive index  $n$ , for the wavelength region in question, if the magnitude of the chromatic aberration is to be assessed.

5) The conjugate parameter  $C$  which is computed from the magnification  $m$  (for that component alone), using

$$C = \frac{m+1}{m-1}$$

The Seidel coefficients then become:

$$S_1 = h^4 K^3 (a_0 + a_1 C + a_2 C^2)$$

$$S_2 = -h^2 H (b_0 + b_1 C)$$

$$S_3 = H^2 K$$

$$S_4 = \frac{H^2 K}{n}$$

$$S_5 = 0$$

$$C_1 = \frac{h^2 K}{V}$$

$$C_2 = 0$$

Equations for a thin lens located away from the stop - Stop shift equations

For lenses located away from the stop, the equations are slightly more complex. They require a knowledge of the height  $h$  at the lens of the paraxial chief ray from the object point of interest. It also passes through the center of the stop. This is incorporated in the term  $E$ , given by

$$E = \frac{\bar{h}}{h}$$

The primed quantities ( $S_1'$  etc.) refer to the coefficients obtained if the lens were at the stop.

$$S_1 = S_1'$$

$$S_2 = S_2' + ES_1'$$

$$S_3 = S_3' + 2ES_2' + E^2 S_1'^2$$

$$S_4 = S_4'$$

$$S_5 = S_5' + E(3S_3' + S_1'^2) + 3E^2 S_2' + E^3 S_1'^3$$

$$C_1 = C_1'$$

$$C_2 = C_2' + EC_1'$$

# Theory

## Aberrations

### Example 7

As an example of the calculation of the approximate performance of a lens system using the equations on the previous Page, we will apply them to the lens combination used in Example 5 on previous page.

For lens 1,  
(from Example 5)  
 $m = u_1/u_0 = 0$ , giving a conjugate parameter  
 $C = -1$ .  
 $h = 7.5$   
 $K = 1/75$   
 $H = -0.75$   
 $h = 0$

(from the lens table on previous page.)

$n = 1.5168$   
 $V = 64.17$   
 $a_0 = 4.3238$   
 $a_1 = 3.2107$   
 $a_2 = 1.0796$   
 $b_0 = 1.6053$   
 $b_1 = 1.3296$

Substituting these values into the equations on the previous page, we obtain

$$\begin{aligned} S_1 &= 0.016441 & S_2 &= 0.002068 \\ S_3 &= 0.0075 & S_4 &= 0.004945 \\ S_5 &= 0.0 & C_1 &= 0.0011688 \\ C_2 &= 0.0 \end{aligned}$$

For lens 2  
(from example 5)

$$\begin{aligned} m &= u_2/u_1 = 2/3, \text{ giving a conjugate parameter} \\ C &= -5. \\ h &= 3.75 \\ K &= 1/75 \\ H &= -0.75 \\ h &= 3.75 \end{aligned}$$

The values of  $n, V, a_0, a_1, a_2, b_0$  and  $b_1$  are the same as for lens 1 as it is identical in material and form.

As this lens is not located at the stop, additional shift values must be applied to the results obtained using the equations for a lens at the stop. If the lens were at the stop we would obtain the following values

$$\begin{aligned} S_1 &= 0.007146 & S_2 &= -0.009455 \\ S_3 &= 0.0075 & S_4 &= 0.004945 \\ S_5 &= 0.0 & C_1 &= 0.002922 \\ C_2 &= 0.0 \end{aligned}$$

For lens 2 the required term in the stop shift equations  $E = \bar{h}/h = 1$ . Using this value we obtain the final contributions from lens 2 as

$$\begin{aligned} S_1 &= 0.007146 & S_2 &= -0.002309 \\ S_3 &= -0.004264 & S_4 &= 0.004945 \\ S_5 &= 0.006226 & C_1 &= 0.002922 \\ C_2 &= 0.002922 \end{aligned}$$

The totals for the system are therefore

$$\begin{aligned} S_1 &= 0.023587 & S_2 &= -0.000241 \\ S_3 &= 0.003236 & S_4 &= 0.00989 \\ S_5 &= 0.006226 & C_1 &= 0.01461 \\ C_2 &= 0.002922 \end{aligned}$$

For this system these values are very close for practical purposes to those obtained from a surface by surface calculation, which includes the effects of lens thickness.

### Interpretation of Seidal Coefficients

Computation of the Seidal coefficients allows us to evaluate aberrations in a system. Different Seidal coefficients indicate the presence of different aberrations. A non-zero value will indicate that the following aberrations are present.

$S_1$  Spherical Aberration  
 $S_2$  Coma  
 $S_3 + S_4$  Field Curvature & Astigmatism  
 $S_5$  Distortion  
 $C_1$  or  $C_2$  Chromatic Aberration

The following brief discussion on the interpretation of these values should assist in assessing the aberrations of a system.

### Spherical aberration

The most important aberration for the majority of systems is Spherical Aberration as it occurs over the whole field. The spot size in the paraxial focal plane is given by

$$S_1/u_{N+1}$$

It can be reduced by a factor of 4 with a suitable refocus of

$$3S_1/8u_{N+1}^2$$

The associated wavefront error is  $S_1/8$ ;

If  $|S_1|$  is less than  $7.6\lambda$  the system is likely to be diffraction limited on - axis (if no higher orders are present).

### Coma

The distance from the chief ray intersection point to the extreme ray in the coma pattern is given by

$$3S_2/2u_{N+1}$$

If  $|S_2|$  is less than  $1.2\lambda$ , then the system should be diffraction limited for coma.

### Astigmatism & field curvature

Off-axis the situation is complicated by the interaction between all the aberrations on the wavefront. For diffraction limited performance look for  $|S_3|$  and  $|S_4|$  to be lower than  $\lambda$ . These figures are not absolute values but indicate when a lens will be completely unsuitable. (See Welford for a more complete treatment.)

The astigmatic blur in the paraxial focal plane is an ellipse with dimensions  $(3S_3 + S_4)/u_{N+1}$  by  $(S_3 + S_4)/u_{N+1}$ .

### Distortion

The distortion of the edges of the image as a fraction of the Gaussian image height is given by  $S_5/2H$ .

### Chromatic aberration

If focused in the paraxial focal plane at one end of the spectral range the blur diameter would be  $2C_1/u_{N+1}$  due to longitudinal chromatic aberration. In simple terms the blur is  $1/V$  times the lens diameter for an infinite object distance.

# Theory

## Doublets

### CEMENTED ACHROMATIC DOUBLETS

Cemented Achromatic Doublets offer greatly improved color correction to single BK7 components. In addition they have superior imaging performance even for monochromatic imagery.

### Ideal conditions with slower doublets

Achromatic Doublets consist of two materials of differing dispersion characteristics. By combining a strong crown component with a weaker flint component of the opposite power, the chromatic aberration of the combination can be considerably reduced.

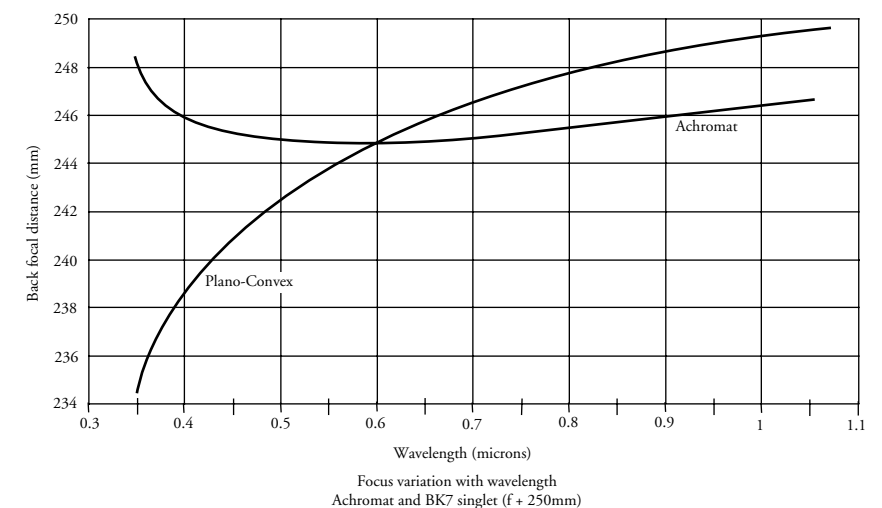
If the ratio of the lens powers of the crown and flint components are chosen to be the same as the ratio of their Abbe numbers or V- values for a particular spectral range, then the paraxial foci for the two extreme wavelengths will coincide.

For the achromats in this catalog the central design wavelength is 587.6nm (d line - yellow) with an achromatization target at 486.1nm (F line - blue) and 656.3nm (C line - red). Typical glass pairings will result in a crown component with approximately 2.5 times the power of the final doublet, and a flint component with approximately - 1.5 times the power of the final doublet.

### Secondary Spectrum

Because of a fundamental property of optical materials there is always a small residual difference between the green focus and the red/blue focus. This phenomenon is known as secondary spectrum. For a typical achromatic doublet this focal shift is 1/2000th of the focal length of the doublet. The focal excursions of an achromatic doublet and a BK7 singlet of equal focal length are plotted in the figure below. Over the visible region (480 - 650nm) of the spectrum the focal shift of an achromatic doublet is approximately 30 times smaller than for a BK7 singlet (Abbe number = 64). In most cases this is acceptably small compared with other remaining residual errors.

The exception, where it becomes the dominant aberration, is in long focal length lenses of low relative aperture e.g. collimator systems or refracting astronomical objectives. This type of lens is often hand - figured to give excellent correction of the monochromatic performance. In addition, for these cases, the cost/performance considerations are different, and more expensive materials and/or a three glass design can be used.



Each of the component parts of the doublet has spherical aberration which varies quadratically with lens bending. As always the positive crown component has undercorrected spherical aberration, while the negative flint component introduces a contribution of the opposite sign. The combination has a variation of spherical aberration which has a quadratic dependence on the shape factor of the doublet, but with greatly reduced magnitude compared with a single component of the same refracting power. Given a particular choice of crown component, it may be possible to choose a flint material which allows the overall spherical aberration to become slightly overcorrected for a range of lens bendings.

Considering the coma contribution of each of the components, one finds that they are of opposite sign and have a linear dependence on lens bending, as does the combination.

By choosing certain glass combinations it is possible to have simultaneously a high degree of correction of Spherical Aberration, Coma and Primary Color. Such a lens is aplanatic as well as achromatic.

# Theory

## Doublets

### High Aperture Doublets and the Significance of Higher Order Aberrations.

As the relative aperture of a doublet is increased the higher order aberrations become significant. Unfortunately there are insufficient degrees of freedom to correct both the higher order aberration and the primary aberrations in a cemented doublet. Within the restrictions of the cemented doublet format the general procedure is to balance the unavoidable higher order aberrations with appropriate amounts of primary aberration.

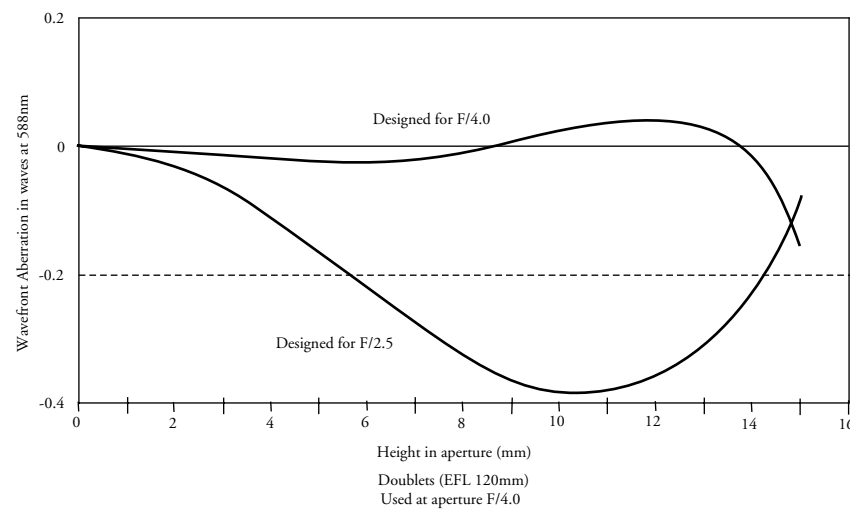
The cemented surface introduces a large amount of overcorrection of the aberrations, by using high angles of incidence with a small difference in refractive index. This mechanism causes the higher order aberrations to be overcorrected in a standard doublet.

The optimum choice to improve the performance is to select undercorrected targets for the following aberrations - Primary Color, Primary Spherical Aberration and Primary Coma.

If a doublet designed for a high relative aperture is stopped down, then the image quality will improve. However the presence of the undercorrected primary aberrations, which it was necessary to introduce, will always make the performance inferior to that of a doublet which was designed for the target aperture. The effect is illustrated by the example of two Ealing doublets of focal length 120mm used at F/4.0, (see figure below.)

For the reasons outlined above, as a rule of thumb, (if the F/No is faster than F/5), it is usually advisable not to select a doublet which is excessively larger than required,

It is also worth noting that the monochromatic aberrations of faster doublets can be significantly improved if a doublet of around 1.5 times the required focal length is combined with a meniscus lens of the aplanatic type to give the desired total power.



# Theory

## Overall System Considerations

### Overall System Considerations

There are many factors which should be considered when constructing an optical system. The Books section at the end of this Catalog has a list of relevant texts, available through Ealing, which deal with these aspects in more detail than can possibly be covered in the following short discussion. It is likely that you may need to consult more than one of these to cover all the areas relevant to your application if it is particularly complex.

A possible list of main criteria which may apply to simple systems that are likely to be constructable from standard components is as follows

- System Packaging Requirements and Field of View
- Spectral Bandwidth and Transmission
- System Resolution and Aberrations
- Environmental

Even if you are not designing the system yourself, you are very likely to be asked questions about these.

### System Packaging Requirements and Field of View

The packaging requirements basically involve squeezing your system into an available volume, with any necessary clearances. This and the Field of View requirements are the predominant, but not the sole, driving factors to selecting the focal lengths and diameters of components.

One way of starting a system design is to examine all the positions at which any images are required to be formed and their magnification, and then to determine the thin lens focal lengths necessary to meet these conditions. This can be achieved by repeated applications of the conjugate equations.

The paraxial raytrace equations are probably the most appropriate equations to use for the second stage analysis once a rough idea of the focal lengths of the individual

modules in your system has been obtained. By tracing two paraxial rays through a chosen system of thin lenses you can determine fairly good approximations of the diameters you are likely to require for each lens, and the field angle and magnification that each is likely to operate at.

If the paraxial raytrace shows up any lens where the diameter is a substantial fraction of its focal length, or where the field angle is more than 5 degrees, then it is likely that more than one component is going to be needed at that point to achieve good resolution. If the field angle approaches 45 degrees, or the diameter equals or exceeds the focal length, then even a complex lens assembly may not offer a solution. Eliminating problem areas like these, where you can, is the way to improve performance and reduce system cost. A paraxial layout is all important here.

If possible try to lay out the optics first and provide enough space afterwards. If size is an a priori requirement then be prepared to consider the possibility of folding your system using mirrors and prisms. As the use of weaker components can have a beneficial effect on performance, the extra length of a folded system may allow them to be used.

### Spectral Bandwidth and Transmission

The choice of materials for the components is usually defined by the spectral bandwidths over which transmission is required. Windows, lenses and beamsplitters (used in transmission) should all be transparent at the required wavelengths. In addition, if you are working with high power lasers, it is often advisable to use substrates which are transparent as well.

Where high power lasers are concerned you should look for materials with the lowest absorption or a high thermal conductivity. Zinc Selenide for the Infrared and Fused

Silica for the Visible and Ultraviolet are the preferred materials. Specially prepared grades of BK7, such as platinum - free, are also used for this purpose.

Lenses and windows made from high refractive index materials should be coated in order to reduce surface reflections. Even where the directly transmitted radiation gives an adequate signal, the surface reflections may produce ghost images or glare which reduces image contrast. The signal to noise ratio can be the more important consideration.

For the most critical applications you may need to include extra filters or stops and baffles, which serve no other purpose than to reduce the unwanted signals which can arise in a system. These can occur from surface or bulk material scatter of the components, or the side walls of the housings. The scatter characteristics of a mock - up on a lens bench can be very different from the prototype lens in a proper housing. If the results from your assembly have changed between these two stages in a project, scatter could be your problem.

### System Resolution and Aberrations

Aberration levels

One of the most complex areas to examine is the aberrations of a system. It is not possible to present all the techniques which are employed in their calculation. Certain equations have been given earlier, to allow initial assessments to be made. You may find them useful, at an early stage, in identifying areas in a system which need something better than a single lens.

There is a discussion of the general forms of the more common aberrations earlier in this Theory section. The equations are based on third order thin lens theory only. Therefore they can only give an approximate indication of the

Optics

Lenses & Microscope Components

Coatings

Mirrors & Beamsplitters

Prisms & Polarizers

Filters

Pinholes

Opto-mechanics

Rails

Mounting Hardware

Mirror & Component Mounts

Manual Micro Positioners

Motorized Positioners

Optical Instruments

Microscopes

Light Sources

Optics

Lenses & Microscope Components

Coatings

Mirrors & Beamsplitters

Prisms & Polarizers

Filters

Pinholes

Opto-mechanics

Rails

Mounting Hardware

Mirror & Component Mounts

Manual Micro Positioners

Motorized Positioners

Optical Instruments

Microscopes

Light Sources

aberrations. However, for many systems they are good enough to determine the overall behavior. For the important cases of Spherical Aberration and Axial Color - the ones most likely to concern typical users of standard catalog components - the stop shift equations are not likely to be required. If you intend doing serious optical design work, then you are strongly advised to buy a book on the subject. (See Section 5.)

Before we go any farther in this discussion it must be pointed out that not all aberrations can be conveniently corrected with systems constructed exclusively of simple stock components. Because this is especially true of systems requiring extended field coverage, we will discuss this first and the reasons why this is so.

### Astigmatism and Field curvature

The most serious aberrations which will limit the off - axis performance of any system using standard components are likely to be Primary Astigmatism and Field Curvature. If any of the components in the system is used at a field angle in excess of a few degrees then it is very likely that these aberrations will occur. Depending on how good the system has to be, they may need attention. They vary as the square of the field angle and can quickly swamp the spherical aberration at off - axis field points. This is particularly noticeable for lenses such as achromatic doublets in which the axial aberrations and Coma have been reduced.

A lens which is required to cover a large angular field of view will require the astigmatism and field curvature to be small if it is to be worth fully correcting the Spherical Aberration, Coma and Axial Color.

If the system consists of a thin lens at the stop then the third order astigmatism and field curvature can be estimated using previous equations. These terms involve three parameters - the refractive index  $n$ ,

the lens power  $K$  and the Lagrange Invariant  $H$ . For a thin lens there are no terms involving the lens shape. Therefore we have no hope of eliminating these aberrations with a single lens at the stop. However if you examine the stop shift equations you will see that the expression for astigmatism contains terms involving the Spherical Aberration and Coma. Clearly a way of compensating for astigmatism is to have lenses with specific non - zero amounts of Spherical Aberration and Coma at locations away from the stop. However as standard singlet lenses are only available in certain shapes, and doublets are designed to have very small amounts of Spherical Aberration and Coma, the task of reducing Astigmatism in a system of stock components is not really a practical option.

Third order Field Curvature is even more difficult to remove as it is completely unaffected by stop shift. A field flattening lens, usually plano - concave with the plane surface towards and very close to the image, can sometimes help. Bear in mind that due to its proximity to the image plane, any scratches or defects on a field flattening lens will almost be in focus.

If you have a system which is currently afflicted with Astigmatism or Field Curvature, or a theoretical analysis suggests it may be significant, then you could consider the following courses of action before resorting to a custom solution or a change in design concept.

1) See if the field angle can be reduced at critical areas by increasing the system length. For example, if you have to image a particular linear object at a set magnification, doubling the length of the system at fixed  $F$ /ratio has no effect on the Lagrange Invariant  $H$ , but halves the value of power  $K$ .

2) You could try replacing critical simple components in the system with complex anastigmatic lenses (e.g. camera or enlarger lenses). This is not a universal panacea, however,

and really only one such lens should ever be contemplated. Information on the entrance and exit pupil positions and the locations of the principal planes is not always readily available, even from the manufacturer. Unless you can match the pupils into the rest of your system you may experience vignetting problems (with loss of off - axis illumination). Although it can seem particularly attractive, it is best to avoid using infinity corrected optics such as camera lenses back - to - back in an attempt to create wide field finite conjugate imaging. The light loss can be significant without using field lenses, which will introduce more of the Field Curvature you were trying to eliminate.

### Axial and Lateral Color

Looking at the previous equations you will see that the Lateral Color of a thin lens at the stop is zero. The Axial Color is proportional to the lens power and inversely proportional to the Abbe number. If a single lens is to be used then it is advisable to select a material with a high Abbe number. We have quoted values of this parameter for certain materials and the major wavebands at various places in the Catalog, and you can determine it for your application using the equation on.

An estimate of the amount of Axial Color contributed by a single lens at the stop can be made using the previous equations. It is possible to sum these for a complete system, including those systems where lenses are located away from the stop. For lenses away from the stop the stop - shift equations will have to be applied to determine the Lateral Color.

Where the value of axial color is unacceptable then there are several courses of action.

1) Reduce the bandwidth, as this will effectively increase the value of the Abbe number. Provided there is sufficient signal available then this

could be an acceptable method.

2) Use a combination of lenses with opposing powers and different Abbe numbers. This technique is used in the cemented achromatic doublets.

3) Consider a reflecting solution. For systems requiring the broadest bandwidths this is often the only option.

A drawback to reflecting systems is the need to either work off - axis or to accept obscuration of the aperture. The conic mirrors are all free of Color and Spherical Aberration. They do however suffer from Coma and the other field aberrations.

In the Infrared region the Abbe numbers of most of the materials are quite high, so achromatization is often not necessary.

In the Ultraviolet region achromatization is possible using Calcium Fluoride and Fused Silica. However, if the wavelength range is too large the secondary spectrum can become dominant and there is no third material available to correct this. Also the components are not usually cemented, which limits the  $F$ /ratios available unless many components are used.

In the visible region, the achromatic cemented doublets offer axial color values around 30 times smaller than BK7 lenses of the same focal length and diameter. However, the cemented surface prevents their use in high power applications.

### Spherical aberration and Coma

If you are using a simple component or a doublet make sure you have chosen the best standard form for your application, and that you are using it the correct way round for its conjugates of usage.

Look for any parts of the system where the axial clear aperture of a

component given by the paraxial raytrace is a significant fraction of its focal length. Consider sharing the labor of that component amongst several components of weaker power (longer focal length).

If your system consists entirely of simple components then you may use the equations on previous pages to make an estimate of the third order (Primary) Spherical Aberration of the system. To assess the Coma of a multi - lens system, you will need to include the "stop shift" terms for any components which are not at the stop.

The relevant coefficients are listed along with each lens material and type. They have been calculated for the design wavelength and assume the lenses are thin. However they are a sufficiently good approximation for estimates of performance to be made and will help you avoid totally unsuitable lens choices. A precise evaluation of lens performance requires real rays to be traced through the system.

If either of these aberrations is too large then you should first consider reducing the numerical aperture by means of a mechanical stop, as this can have a significant effect on both the Spherical Aberration and Coma. If the aperture cannot be reduced because of the need to maintain light throughput or to give adequate resolving power, then a change in construction will be necessary.

### Environmental

The components nearest to high power lamps can often be exposed to considerable amounts of thermal radiation; Fused Quartz or Fused Silica is preferable to ordinary glass in these areas. If the thermal radiation is not required further down the system you should always try to remove it by using Hot/Cold mirrors, according to the geometry of your system or, if the power levels are lower, with a heat absorbing glass such as the Schott KG glasses.

Certain materials have a refractive index which varies strongly with temperature, e.g. Germanium. For critical applications you may need to consider passive compensation by designing the appropriate mounts, or active compensation by using microprocessor control movements (e.g. Ealing DPS equipment).

For applications such as laser cutting, where hot debris can be thrown back onto the optical components, consider the use of sacrificial windows to limit the damage. If the wavelength permits, the sapphire windows are both durable and able to withstand rigorous cleaning. Another approach is to make the light reach the workpiece by way of a fold mirror, keeping the more expensive lens components safely out of the line of fire of debris from the cutting area.